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MATHEMATICS

A RING OF FUNCTIONS ON A TOPOLOGICAL GROUP

By MARK KREIN

(Communicated by A. N. Kolmogoroff, Member of the Academy, 9.IX.1949)

1. Let G be a topological group. A function $f(g)$ $(g \in G)$ which assumes, generally speaking, complex values, is called positive definite (abbreviated p. d.) if $f(g)$ is a Hermitian function, i. e. $f(g^{-1}) = \overline{f(g)}$ * $(g \in G)$ and for any $g_1 \in G$, $g_2 \in G$, ..., $g_n \in G$ $(n = 1, 2, \ldots)$ the Hermitian form

$$\sum_{j,k=1}^{n} f(g_j g_k^{-1})\, \xi_j \bar{\xi}_k \tag{1}$$

is non-negative. The non-negativity of the form (1) for $n = 2$ means nothing else than

$$|f(g)| \leqslant f(e) \quad (g \in G, \quad e = gg^{-1}). \tag{2}$$

Assuming in (1) $n = 3$, $\xi_1 = 1$, $\xi_2 = \eta$, $\xi_3 = -\eta$, $g_1 = e$, $g_2 = g$, $g_3 = h$, we obtain

$$f(e) + 2R\{(f(g) - f(h))\bar{\eta}\} + 2[f(e) - R\{f(gh^{-1})\}]|\eta|^2 \geqslant 0$$

where $R\{z\}$ denotes the real part of the number z. Since η is in this inequality an arbitrary complex number, it follows from it that **

$$|f(g) - f(h)|^2 \leqslant 2f(e)[f(e) - R\{f(gh^{-1})\}] \quad (g,\ h \in G). \tag{3}$$

The latter inequality shows that if a p. d. function $f(g)$ is continuous at one point $g = e$, only then it is uniformly continuous on G***.

Denote by $\mathfrak{P}$ the set of all continuous p. d. functions and by $\mathfrak{R}$ their complex linear envelope. Evidently, a certain function $\varphi(g)$ $(g \in G)$ belongs to $\mathfrak{R}$ if and only if each of its Hermitian components

$$\varphi^+(g) = \frac{1}{2}\{\varphi(g) + \overline{\varphi(g^{-1})}\}, \qquad \varphi^-(g) = \frac{1}{2i}\{\varphi(g) - \overline{\varphi(g^{-1})}\} \tag{4}$$

is representable as the difference of two functions from $\mathfrak{P}$.

The set $\mathfrak{R}$ is a ring, since by a known algebraical theorem of J. Schur (3), the product of two functions from $\mathfrak{P}$ belongs again to $\mathfrak{P}$.

---

* As usual the bar denotes transition to the complexly conjugate value.
** Compare with Lemma 1 of the paper of D. Raïkov (1).
*** This remark is a generalization of a remark of A. Artemenko (2).

If a Hermitian function $\varphi(g) \in \mathfrak{R}$, then let us put

$$\|\varphi\| = \inf [f^{(1)}(e) + f^{(2)}(e)] \tag{5}$$

where the infimum is taken over all pairs $f^{(i)}(g) \in \mathfrak{P}$ $(i = 1, 2)$ such that $\varphi = f^{(1)} - f^{(2)}$; in particular,

$$\|\varphi\| = \varphi(e) \qquad \text{for } \varphi \in \mathfrak{P}.$$

For an arbitrary function $\varphi \in \mathfrak{R}$ we put

$$\|\varphi\| = \sup_{\alpha} \|\varphi^+ \cos\alpha + \varphi^- \sin\alpha\| \tag{6}$$

where φ^+ and φ^- are Hermitian components of the function φ.

It is easily seen that the norm $\|\varphi\|$ $(\varphi \in \mathfrak{R})$ defined in this way possesses the following properties:

1°. $\|\varphi\| > 0$, if $\varphi \not\equiv 0$;
2°. $\|\lambda\varphi\| = |\lambda| \, \|\varphi\|$; ($\lambda$ is a complex number)
3°. $\|\varphi + \psi\| \leqslant \|\varphi\| + \|\psi\|$;
4°. $\|\varphi\psi\| \leqslant \sqrt{2} \, \|\varphi\| \, \|\psi\|$ $(\varphi, \psi \in \mathfrak{R})$

and the factor $\sqrt{2}$ in 4° can be omitted, if one of the functions φ and ψ is Hermitian. Besides, from (2) and (5) follows, that if $\varphi \in \mathfrak{R}$ is a Hermitian function, then

$$\sup_{g \in G} |\varphi(g)| \leqslant \|\varphi\|. \tag{7}$$

Theorem 1. *The ring* $\mathfrak{R}$ *is complete by the norm* $\|\varphi\|$.

We omit the simple proof of this theorem, since in the next Note we prove a more general theorem.

2. Replace the topology given in G by the discrete topology. Then the rôle of the ring $\mathfrak{R}$ will be played by the ring $\mathfrak{R}'$, which is the linear complex envelope of the set of all p. d. functions. Construct for the elements of $\mathfrak{R}'$ the norm $\|\varphi\|'$ in the same way as it has been done for the elements of $\mathfrak{R}$. There arises the following problem. *Does* $\mathfrak{R}$ *represent the set of all continuous functions from* $\mathfrak{R}'$, *and if* $\varphi \in \mathfrak{R}$, *does the equality* $\|\varphi\| = \|\varphi\|'$ *hold*? This problem has a positive solution if and only if the following theorem holds:

Theorem 2. *If a certain Hermitian continuous function* $\varphi(g)$ $(g \in G)$ *admits of the representation*

$$\varphi(g) = F^{(1)}(g) - F^{(2)}(g) \qquad (g \in G) \tag{8}$$

where $F^{(i)}$ $(i = 1, 2)$ *are p. d. functions, then it admits also of the representation*

$$\varphi(g) = f^{(1)}(g) - f^{(2)}(g) \qquad (g \in G) \tag{9}$$

where $f^{(i)}$ $(i = 1, 2)$ *are continuous p. d. functions and* $f^{(1)}(e) + f^{(2)}(e) \leqslant F^{(1)}(e) + F^{(2)}(e)$.

In the sequel we prove this theorem in the following two cases: 1) G is a compact group, 2) G is an arbitrary commutative group. In these two cases it also follows from our proof of Theorem 2 that the functions $f^{(1)}$ and $f^{(2)}$ can be chosen so that

$$\|\varphi\| = f^{(1)}(e) + f^{(2)}(e).$$

3. Proof of Theorem 2 for Any Compact Group. In this case we have to use the theory of Peter-Weyl [see L. Pontrjagin (4)]. Let us consider the integral equation

$$\psi(g) - \lambda \int_G \varphi(gh^{-1}) \, \psi(h) \, dh \qquad \left(\int_G dh = 1\right). \tag{10}$$

Let

$$\varphi(gh^{-1}) \sim \sum_{k=1}^{\infty} \frac{\psi_k(g)\,\overline{\psi_k(h)}}{\lambda_k} \tag{11}$$

be a formal expansion of the kernel $\varphi(gh^{-1})$ in the bilinear series of fundamental functions ψ_k $(k=1, 2, \ldots)$ of the integral equation (10). For the proof of the theorem it is sufficient to prove that from (8) follows

$$S = \sum_{k=1}^{\infty} \frac{1}{|\lambda_k|} \leqslant F_1(e) + F_2(e). \tag{12}$$

In fact, from the finiteness of the quantity S it is easy to conclude, using the well-known properties of the expansion (11), that it converges absolutely and uniformly. Then assuming

$$f^{(1)}(gh^{-1}) = \sum_{\lambda_k>0} \frac{\psi_k(g)\,\overline{\psi_k(h)}}{\lambda_k}; \quad f^{(2)}(gh^{-1}) = -\sum_{\lambda_k<0} \frac{\psi_k(g)\,\overline{\psi_k(h)}}{\lambda_k}$$

we shall have $f^{(i)}(g) \in \mathfrak{P}$ $(i=1, 2)$, $\varphi(g) = f^{(1)}(g) - f^{(2)}(g)$ $(g \in G)$ and, besides

$$f^{(1)}(e) + f^{(2)}(e) = \sum_{k=1}^{\infty} \frac{1}{|\lambda_k|}. \tag{13}$$

Let us show that from (8) follows (12). Let g_j $(j=1, 2, \ldots, n)$ be, for the present, some arbitrary elements of G. Denote by $\rho_1^{(i)} \geqslant \rho_2^{(i)} \geqslant \ldots$ $\ldots \geqslant \rho_n^{(i)}$ $(\geqslant 0)$ $(i=1, 2)$ the characteristic values (Eigenwerte) of the matrix $\left\| \frac{1}{n} F^{(i)}(g_j g_k^{-1}) \right\|$ $(j, k=1, 2, \ldots, n)$, i. e. the roots of the equation $\left| \frac{1}{n} F^{(i)}(g_j g_k^{-1}) - \rho\delta_{jk} \right| = 0$. On the other hand, let $\sigma_1 \geqslant \sigma_2 \geqslant \ldots \geqslant \sigma_p$ $(\geqslant 0)$ be all the non-negative, and $\tau_1 \leqslant \tau_2 \leqslant \ldots \leqslant \tau_q$ (< 0) all the negative characteristic values of the matrix $\left\| \frac{1}{n} \varphi(g_j g_k^{-1}) \right\|$ $(j, k=1, 2, \ldots, n)$. Since, in virtue of (8)

$$-\sum_{j,k=1}^{n} F^{(2)}(g_j g_k^{-1})\,\xi_j \bar{\xi}_k \leqslant \sum_{j,k=1}^{n} \varphi(g_j g_k^{-1})\,\xi_j \bar{\xi}_k \leqslant \sum_{j,k=1}^{n} F^{(1)}(g_j g_k^{-1})\,\xi_j \bar{\xi}_k$$

from the well-known properties of the characteristic values [3] follows that

$$\sigma_k \leqslant \rho_k^{(1)} \quad (k=1, 2, \ldots, p), \quad -\tau_k \leqslant \rho_k^{(2)} \quad (k=1, 2, \ldots, q)$$

whence

$$s_n = \sum_{1}^{p} \sigma_k + \sum_{1}^{q} |\tau_k| \leqslant \sum_{k=1}^{n} \rho_k^{(1)} + \sum_{k=1}^{n} \rho_k^{(2)} = F^{(1)}e + F^{(2)}(e). \tag{14}$$

But with a corresponding choice of q_i $(i=1, 2, \ldots, n)$ and $n \to \infty$ we can make the determinant

$$\Delta_n(\lambda) = \left| \delta_{jk} - \frac{\lambda}{n} \varphi(g_j g_k^{-1}) \right|_1^n$$

converge uniformly in every finite part of the plane of the complex variable to a determinant of Fredholm $D(\lambda)$ of the integral equation (10). But then s_n, being the sum of absolute values of the characteristic

number $\Delta_n(\lambda)$, converges to S. Hence from (14) follows (12), which proves the theorem.

Observe that together with the theorem we have proved that the group G being compact, for any Hermitian function $\varphi \in \mathfrak{R}$ the norm $\|\varphi\|$ can be calculated by the formula [which follows from comparison of (12) and (13)]:

$$\|\varphi\| = \sum_{k=1}^{\infty} \frac{1}{|\lambda_k|}.$$

4. Now let G be an arbitrary commutative group. In this case we have to introduce several preliminary conceptions and remarks.

Denote by X any sufficiently complete * group of characters $\chi(g)$ $[|\chi(g)|^2 = \chi(g)\chi(g^{-1}) = 1, \ \chi(gh) = \chi(g)\chi(h), \ g, h \in G]$ of the abstract group G. To every $g \in G$ there corresponds a function on X: $g(\chi) = \chi(g)$ $(\chi \in X)$. We introduce for the function $P(\chi)$ defined on X the norm

$$\|P\| = \sup_{\chi \in X} |P(\chi)|.$$

Denote by B the closure with respect to the norm $\|P\|$ of the linear complex envelope of all possible functions $g(\chi)$ $(\chi \in X)$. Evidently, B is a ring, and consequently, if $P(g) \in B$, then any polynomial of $P(g)$ enters into B, and therefore $|P(g)| \in B$ as well.

A linear (i. e. additive and continuous) functional $\Phi(P)$ $(P \in B)$ is said to be r e a l, if it takes real values on real functions $P(\chi) \in B$; if, besides, $\Phi(P) \geqslant 0$ for $P(\chi) \geqslant 0$ $(\chi \in X)$, then the functional $\Phi(P)$ is said to be p o s i t i v e. It is easily seen that the norm of the real functional Φ does not change, if we consider Φ not on B, but on the aggregate B' of all real functions from B'. Now note also, that if $\Psi(P)$ $(P \in B)$ is a linear positive functional, then

$$\|\Psi\| = \sup_{P \in B} \frac{|\Psi(P)|}{\|P\|} = \Psi(1) \qquad (15)$$

and the inequality of Schwarz

$$|\Psi(PQ)|^2 \leqslant \Psi(|P|^2)\,\Psi(|Q|^2) \qquad (P, Q \in B) \qquad (16)$$

holds.

From the fact that $e(\chi) \equiv 1 \in B'$ and from (15) follows that any real functional Φ admits of the decomposition

$$\Phi(P) = \Phi_+(P) - \Phi_-(P) \qquad (P \in B) \qquad (17)$$

where Φ_+ and Φ_- are positive functionals such that

$$\|\Phi\| = \|\Phi_+\| + \|\Phi_-\|.$$

Besides, from the fact that if $P \in B$ then also $|P| \in B$, follows (6), that for any $\varepsilon > 0$ there exists a decomposition of the unity:

$$1 = u_1(\chi) + u_2(\chi), \quad u_i(\chi) \geqslant 0 \quad (\chi \in X), \quad u_i \in B' \quad (i = 1, 2) \qquad (18)$$

such that

$$0 \leqslant \Phi_+(u_2) < \varepsilon, \qquad 0 \leqslant \Phi_-(u_1) < \varepsilon. \qquad (19)$$

* It can be proved by the same method that if a continuous Hermitian kernel $K(s, t)$ $(a \leqslant s, t \leqslant b)$ is representable as the difference of two bounded Hermitian positive kernels, the sum of the absolute values of the inverse characteristic numbers of $K(s, t)$ converges.

Let us show that with such a choice of functions $u_1(\chi)$ and $u_2(\chi)$ the inequality

$$|\Phi_+(P) - \Phi(u_1 P)| \leqslant 2\sqrt{\varepsilon \|\Phi\|}\ \|P\| \tag{20}$$

holds. In fact, in virtue of Schwarz' inequality and the fact that $u_1^2(\chi) \leqslant u_1(\chi)$ $(\chi \in \mathfrak{X})$ and also (19) we have

$$|\Phi_+(P) - \Phi_+(u_1 P)|^2 = |\Phi_+(u_2 P)|^2 \leqslant$$
$$\leqslant \Phi_+(u_2^2)\,\Phi_+(|P|^2) \leqslant \Phi_+(u_2)\,\|\Phi_+\|\,\|P\|^2 < \varepsilon\,\|\Phi\|\,\|P\|^2. \tag{21}$$

On the other hand, we find similarly

$$|\Phi_+(u_1 P) - \Phi(u_1 P)|^2 = |\Phi_-(u_1 P)|^2 < \varepsilon\,\|P\|\,\|P\|^2. \tag{22}$$

Comparing (21) and (22) we obtain (20).

5. **Proof of Theorem 2 for Any Commutative Group.** As Raikov (7) has recently proved, *a function $F(g)$ $(g \in G$, G being a commutative group) is positive definite, if and only if there exists a linear positive functional $\Psi(P)$ $(P \in B)$ such that*

$$F(g) = \Psi(g(\chi)) \quad (g \in G).$$

Therefore a Hermitian function $\varphi(g)$ $(g \in G)$ admits of the decomposition (8), if and only if there exists a linear real functional $\Phi(P)$ $(P \in B)$ such that

$$\varphi(g) = \Phi(g(\chi)) \qquad (g \in G),$$

and to every decomposition (8) of the function φ corresponds a decomposition of Φ into the difference of linear positive functionals Φ' and Φ'' and conversely.

Let us consider the decomposition (17). We put

$$f^{(1)}(g) = \Phi_+(g(\chi)), \qquad f^{(2)}(g) = \Phi_-(g(\chi)). \tag{23}$$

Since $F^{(1)}(e) + F^{(2)}(e) = \|\Phi'\| + \|\Phi''\| \geqslant \|\Phi\|$, the quantity

$$f^{(1)}(e) + f^{(2)}(e) = \|\Phi_+\| + \|\Phi_-\| = \Phi\|$$

gives norm $\|\varphi\|$. For the proof of Theorem 2 we have yet to show that if the function $\varphi(g)$ is continuous, then so are also the functions $f^{(i)}(g)$ $(i = 1, 2)$. To this end take an arbitrary $\varepsilon > 0$ and choose for it the functions $u_i(\chi)$ $(i = 1, 2)$ so that the conditions (18) and (19) are satisfied. It is obvious that we can choose $u_1(\chi)$ in the form of a certain «polynomial»

$$u_1(\chi) = \sum_{k=1}^{n} c_k g_k(\chi).$$

Put

$$f_\varepsilon(g) = \Phi[u_1(\chi)\, g(\chi)] = \sum_{k=1}^{n} c_k \Phi[g_k(\chi)\, g(\chi)] =$$

$$= \sum_{k=1}^{n} c_k \varphi(g_k g) \qquad (g \in G). \tag{24}$$

Evidently, $f_\varepsilon(g)$ $(g \in G)$ is a continuous function. On the other hand, it follows from (20), (23) and (24) that

$$|f^{(1)}(g) - f_\varepsilon(g)| \leqslant 2\sqrt{\varepsilon}\,\sqrt{\|\varphi\|} \qquad (g \in G).$$

Since $\varepsilon > 0$ is arbitrary, we conclude that $f^{(1)}(g)$ and therefore $f^{(2)}(g)$ are also continuous functions. Thus, Theorem 2 is proved.

Observe also that in virtue of the known principles of the theory of linear functionals [(8), p. 57] from the cited result of D. Raikov and our Theorem 2 results the following characteristics of the ring $\mathfrak{R}$.

Theorem 3*. *In order that a function* $\varphi(g)$ $(g \in G)$, *where* G *is a commutative group, should belong to the ring* $\mathfrak{R}$, *it is necessary and sufficient that the following two conditions should be satisfied:*

1°. $\varphi(g)$ *is continuous.*

2°. *There exists a constant* M *such that for any* $g_1 \in G, \ldots, g_n \in G$ *and complex numbers* $c_1, \ldots, c_n$ $(n = 1, 2, \ldots)$ *the inequality*

$$|c_1\varphi(g_1) + \ldots + c_n\varphi(g_n)| \leqslant M \sup_{\chi \in X} |c_1 g_1(\chi) + \ldots + c_n g_n(\chi)| \qquad (25)$$

holds.

6. Let now G be a locally compact commutative group satisfying the first axiom of countability and X—a topological group of all continuous characters of the group [(4), ch. V]. In this case holds

Theorem 4. *In order that a function* $\varphi(g)$ $(g \in G)$ *should admit of the representation*

$$\varphi(g) = \int_X g(\chi)\, d\mu(\chi) \qquad (26)$$

where $\mu(E)$ *is a complex-valued absolutely additive function of the Borel sets* $E \subset X$, *it is necessary and sufficient that the function* $\varphi(g)$ *should satisfy the conditions* 1° *and* 2° *of Theorem* 3.

The necessity of the conditions 1°, 2° follows immediately from (26). To prove their sufficiency we can, in virtue of Theorem 3, confine ourselves to the case only, when $\varphi(g) \in \mathfrak{P}$. But in this case the possibility of the representation (26) has been recently proved by A. Powzner (9) and D. Raikov (10) **.

Theorem 3 is generalization of S. Bochner's theorem which states the same thing as Theorem 4 in the particular case, when G is the real axis.

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* If we omit in Theorem 3 the condition 1°, we obtain the characteristics of the ring $\mathfrak{R}'$; the smallest value of the constant M in (25) gives $\|\varphi\|'$ if φ is a Hermitian function. Hence we easily conclude that the supremum figuring in (25) does not depend on the choice of the group X (which can be also asserted on ground of many other reasons).

** The author became familiar with the paper of D. Raikov, after this Note has bee n written. The paper of D. Raikov enables us to state Theorem 3 in a more general and complete form.

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MATHÉMATIQUES

PROBLÈMES AUX LIMITES FONDAMENTAUX DE L'ÉQUATION DES ONDES

Par S. G. MICHLIN

(Présenté par S. L. Soboleff, de l'Académie, le 4.VII.1940)

§ 1. Soit, dans le plan XOY, une courbe L analytique, convexe, sans points singuliers limitant un domaine plan B. Considérons dans le demi-espace (x, y, z), $z \geqslant 0$, un domaine D limité par le domaine B et par un cylindre G, dont la directrice est la courbe envisagée à l'instant et dont la génératrice est parallèle à l'axe OZ. Posons le problème suivant: trouver une solution de l'équation des ondes

$$\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial z^2}=0 \tag{1}$$

satisfaisant dans le domaine B aux conditions initiales

$$U(x, y, 0)=F(x, y), \quad U_z(x, y, 0)=\varphi(x, y) \tag{2}$$

et satisfaisant sur le cylindre G aux conditions aux limites

$$U=\psi(x, y, z) \tag{3_1}$$

où

$$\frac{\partial U}{\partial n}=\chi(x, y, z). \tag{3_2}$$

Posons

$$V=\ln\left(\frac{z_0-z}{r}+\sqrt{\frac{(z_0-z)^2}{r^2}-1}\right), \qquad r^2=(x_0-x)^2+(y_0-y)^2 \tag{4}$$

V est la solution fondamentale de l'équation des ondes considérée par M. Volterra.

Soit σ' une aire, déterminée sur la surface du cylindre G par le cône des caractéristiques

$$z_0-z=\eta. \tag{5}$$

Si le sommet (x_0, y_0, z_0) du cône (5) est situé sur le cylindre, nous désignerons l'aire correspondante σ' par σ_0.

§ 2. Considérons des intégrales, analogues aux potentiels, définies dans le demi-espace $z \geqslant 0$:

$$W_1(x_0, y_0, z_0)=\frac{1}{2\pi}\int\!\!\int_{\sigma'} P(M)\frac{\partial V}{\partial z_0}\,dS=\frac{1}{2\pi}\int\!\!\int_{\sigma'}\frac{P(M)\,dS}{\sqrt{(z_0-z)^2-r_0^2}}, \tag{6}$$

$$W_2(x_0, y_0, z_0)=\frac{1}{2\pi}\int\!\!\int_{\sigma'} Q_z(M)\frac{\partial V}{\partial n}\,dS \tag{7}$$

n étant le sens de la normale extérieure à G.

Si le cône (5) n'a pas de points communs avec le cylindre G dans D, c'est-à-dire si l'aire σ' n'existe pas, nous poserons par définition $W_1 \equiv 0$, $W_2 \equiv 0$. Quand aux fonctions P et Q, nous admetterons qu'elles sont continues sur G et qu'elles possèdent un nombre suffisant de dérivées relativement à z et à u, où u désigne la longueur de l'arc de la courbe L. Admettons encore que ces fonctions s'annullent pour $z = 0$.

Pour les fonctions (6) et (7) ont lieu les théorèmes suivants:

Théorème 1. *Dans le domaine D et dans son complémentaire, les fonctions W_1 et W_2 sont continues et admettent des dérivées du premier ordre continues. Ces fonctions, dans les deux domaines indiqués, vérifient l'équation des ondes* (1) *et satisfont aux conditions initiales*

$$W_1(x, y, 0) = W_{1z}(x, y, 0) = 0; \quad W_2(x, y, 0) = W_{2z}(x, y, 0) = 0.$$

Théorème 2. *Pour tout $M_0 \in G$ on a les égalités suivantes:*

$$\left.\begin{aligned} \frac{\partial W_1^{(i)}(M_0)}{\partial n} &= \frac{1}{2} P(M_0) + \frac{1}{2\pi} \iint_{\sigma_0} P_z(M) \frac{\partial V}{\partial n_0} dS, \\ \frac{\partial W_1^{(e)}(M_0)}{\partial n_0} &= -\frac{1}{2} P(M_0) + \frac{1}{2\pi} \iint_{\sigma_0} P_z(M) \frac{\partial V}{\partial n_0} dS. \end{aligned}\right\} \tag{8}$$

$$\left.\begin{aligned} W_2^{(i)}(M_0) &= -\frac{1}{2} Q(M_0) + \frac{1}{2\pi} \iint_{\sigma_0} Q_z(M) \frac{\partial V}{\partial n} dS, \\ W_2^{(e)}(M_0) &= \frac{1}{2} Q(M_0) + \frac{1}{2\pi} \iint_{\sigma_0} Q_z(M) \frac{\partial V}{\partial n} dS. \end{aligned}\right\} \tag{9}$$

n_0 désignant ici le sens de la normale extérieure à G au point M_0. Les indices i et e indiquent qu'il faut passer à la limite de l'intérieur, respectivement, de l'extérieur de D.

§ 3. Sur le cylindre G prenons z et u pour coordonnées. Alors G sera représenté sur une partie du plan (u, z). L'aire σ_0 sera représentée sur un triangle curviligne (que nous désignerons aussi par σ_0), l'un des côtés duquel se trouve sur l'axe u. Les deux autres côtés d'un tel triangle, qui ont pour sommet un point N, seront appelés $N\alpha_1$-courbes.

On peut démontrer que le point M_0 est un point d'inflexion de la $M_0\alpha_1$-courbe. Les tangentes au point d'inflexion à ces courbes forment avec l'axe u des angles de 45° et, respectivement, de 135°. Au-dessous du point M_0 les courbes passent au-dessus de leurs tangentes, tandis qu'au-dessus du point M_0 elles passent au-dessous de leurs tangentes. La $M_0\alpha_1$-courbe, dont la tangente au point M_0 forme avec l'axe u un angle de 45° (de 135°), sera appelée $M_0\alpha_1$-courbe gauche (droite). Pour z_0 suffisamment petit, ce que nous supposerons toujours dans la suite, la branche supérieure de la $M_0\alpha_1$-courbe tourne sa concavité en bas et la branche inférieure tourne sa concavité en haut.

Considérons un ensemble dénombrable de courbes $M_0\alpha_k$ défini de la façon suivante. Supposons que les lignes $M_0\alpha_\nu$ sont définies pour $\nu < k$. La branche inférieure de la courbe $M_0\alpha_k$ gauche (droite) sera définie comme lieu de points N tels, que la branche supérieure de la courbe $N\alpha_{k-1}$ gauche (droite) soit tangente à la branche inférieure de la $M_0\alpha_1$-courbe gauche (droite). La branche supérieure de la courbe $M_0\alpha_k$ se définit d'une façon analogue.

On peut démontrer que les $M_0\alpha_k$-courbes jouissent des propriétés suivantes:

1. Toute courbe $M_0\alpha_k$ passe par le point M_0; elle est située entre la courbe $M_0\alpha_{k-1}$ et la tangente au point M_0, commune à ces deux courbes. Cette tangente est la position limite, pour $k \to \infty$, des courbes $M_0\alpha_k$.

2. La branche inférieure de la courbe $M_0\alpha_k$ gauche (droite) est le lieu de points N jouissant de cette propriété, que la branche supérieure de la courbe $N\alpha_p$ gauche (droite), $1 \leqslant p \leqslant k-1$, est tangente à la branche inférieure de la courbe $M_0\alpha_{k-p}$ gauche (droite). Une propriété analogue subsiste pour la branche supérieure.

3. Toutes les $M_0\alpha_k$-courbes ont un même sens de concavité.

4. L'angle formé par la $M_0\alpha_k$-courbe gauche (droite) avec l'axe u tend uniformément, pour $k \to \infty$, vers $\frac{\pi}{4}\left(-\frac{\pi}{4}\right)$.

§ 4. Considérons nos problèmes aux limites. La condition (3) étant remplie, nous allons chercher une solution de la forme

$$U(x_0, y_0, z_0) = -\frac{1}{2\pi}\frac{\partial}{\partial z_0}\iint_{S'}[FV_z - \varphi V]\,dS + \frac{1}{2\pi}\iint_{\sigma'} Q_z(M)\frac{\partial V}{\partial n}\,dS \quad (10)$$

et si c'est la condition (3_2) qui a lieu, alors nous chercherons une solution de la forme

$$U(x_0, y_0, z_0) = -\frac{1}{2\pi}\frac{\partial}{\partial z_0}\iint_{S'}[FV_z - \varphi V]\,dS + \frac{1}{2\pi}\iint_{\sigma'}\frac{P(M)\,dS}{\sqrt{(z_0 - z)^2 - r^2}} \quad (10_1)$$

où S' désigne la portion du domaine B découpée par le cône (5). Les fonctions (10) et (10_1) vérifient l'équation des ondes aux conditions initiales. Et pour satisfaire aux conditions aux limites, il suffit de déterminer les fonctions $P(M)$ et $Q(M)$ par les équations

$$Q(M_0) - \frac{1}{\pi}\iint_{\sigma_0} Q_z(M)\frac{\partial V}{\partial n}\,dS = A(M_0). \quad (11)$$

$$P(M_0) + \frac{1}{\pi}\iint_{\sigma_0} Q_z(M)\frac{\partial V}{\partial n}\,dS = C(M_0) \quad (11_1)$$

où $A(M_0)$ et $C_*(M_0)$ sont des fonctions connues. L'équation (11_1) est tout-à-fait analogue à l'équation (11); nous pouvons donc nous limiter à la considération de l'équation (11).

Appliquons à cette équation la méthode habituelle d'itération. Après avoir effectué des transformations nécessaires, nous obtiendrons une équation intégro-différentielle de la forme

$$Q(M_0) - \iint_{\sigma_0} I(M_0, M)\,Q_{zz}(M)\,dS = E(M_0) \quad (12)$$

où le noyau I jouit des propriétés suivantes:

1) Le noyau I est une fonction continue dans le triangle σ_0, s'annulant sur les $M_0\alpha_1$-courbes.

2) La dérivée I_z est continue dans le triangle fermé, limité par les courbes $M_0\alpha_2$ et l'axe u; à l'intérieur des triangles, limités par l'axe u et par les courbes $M_0\alpha_1$ et $M_0\alpha_2$ gauches, respectivement, droites, cette dérivée est continue, mais elle devient infinie sur les $M_0\alpha_2$-courbes. Sur les $M_0\alpha_1$-courbes on a $I_z = 0$.

3) I_z est absolument intégrable dans le triangle σ_0. On peut mettre le noyau I_z sous la forme

$$I_z = \frac{A_1(M_0, M)}{u_0 - u}\sqrt{\left|\frac{\Phi_1(M_0, M)}{\Phi_2(M_0, M)}\right|} + \text{fonction continue} \quad (13_1)$$

à condition que le point M soit situé entre les courbes gauches $M_0\alpha_1$ et $M_0\alpha_2$; A_1 y désigne une fonction analytique et

$$\Phi_k(M_0, M) = z_0 - z - Z_k(u_0, u) \leqslant 0 \quad (14_1)$$

est l'équation de la courbe $M_0\alpha_k$ gauche. Si le point M est situé entre les courbes droites $M_0\alpha_1$ et $M_0\alpha_1$, alors

$$I_1' = \frac{A_1'(M_0, M)}{u_0 - u} \sqrt{\left| \frac{\Psi_1(M_0, M)}{\Psi_2(M_0, M)} \right|} + \text{fonction continue} \qquad (13_2)$$

où A_1' est aussi une fonction analytique, et

$$\Psi_k(M_0, M) = z_0 - z + Z_k(u_0, u) = 0 \qquad (14_2)$$

est l'équation de la courbe $M_0\alpha_k$ droite.

On peut aussi démontrer que

$$I_{z_0} = \frac{B_1(M_0, M)}{u_0 - u} \sqrt{\left| \frac{\Phi_1(M_0, M)}{\Phi_2(M_0, M)} \right|} + \text{fonction continue} \qquad (15_1)$$

si le point M est situé entre les courbes gauches $M_0\alpha_1$ et $M_0\alpha_2$, et que

$$I_{z_1} = \frac{B_1'(M_0, M)}{u_0 - u} \sqrt{\left| \frac{\Psi_1(M_0, M)}{\Psi_2(M_0, M)} \right|} + \text{fonction continue} \qquad (15_2)$$

si M est situé entre les courbes droites $M_0\alpha_1$ et $M_0\alpha_2$. B_1 et B_1' sont des fonctions analytiques. I_{z_0} est continue dans le triangle limité par l'axe et par les courbes $M_0\alpha_2$.

§ 5. Supposons que le noyau $K_m(M_0, M)$ est défini de la façon suivante:

1. $$K_m(M_0, M) = \frac{A_m(M_0, M)}{u_0 - u} \sqrt{\left| \frac{\Phi_{2m-2}(M_0, M)}{\Phi_{2m}(M_0, M)} \right|} \qquad (16_1)$$

si le point M est situé entre les courbes $M_0\alpha_{2m-2}$ et $M_0\alpha_{2m}$.

2. $$K_m(M_0, M) = \frac{A_m'(M_0, M)}{u_0 - u} \sqrt{\left| \frac{\Psi_{2m-2}(M_0, M)}{\Psi_{2m}(M_0, M)} \right|} \qquad (16_2)$$

si le point M est situé entre les courbes $M_0\alpha_{2m-2}$ et $M_0\alpha_{2m}$; A_m et A_m' sont des fonctions analytiques.

3. $$K_m(M_0, M) = 0 \qquad (16_3)$$

dans la partie restée du triangle σ_0*.

Pour $m = 1$ il faut remplacer $2m-2$ par 1. Posons

$$\left.\begin{aligned} I(Q) &= \iint_{\sigma_0} Q_z(M)\, I_z(M_0, M)\, dS, \\ K_m(Q) &= \iint_{\sigma_0} Q_z(M)\, K_m(M_0, M)\, dS. \end{aligned}\right\} \qquad (17)$$

On peut faire voir que

$$K_m I(Q) = \iint_{\sigma_0} \overline{K}_m(M_0, M)\, Q_z(M)\, dS. \qquad (18)$$

La fonction $\overline{K}_m(M_0, M)$ est continue et différentiable, si le point M est situé entre la courbe gauche $M_0\alpha_{2m+2}$ et la courbe droite $M_0\alpha_{2m+2}$. Ensuite

$$\overline{K}_m(M_0, M) = \frac{C_m(M_0, M)}{u_0 - u} \sqrt{\left| \frac{\Phi_1(M_0, M)}{\Phi_{2m+2}(M_0, M)} \right|} + \text{fonction continue} \qquad (19_1)$$

si le point M est situé entre les courbes gauches $M_0\alpha_1$ et $M_0\alpha_{2m+2}$, et

$$\overline{K}_m(M_0, M) = \frac{C_m'(M_0, M)}{u_0 - u} \sqrt{\left| \frac{\Psi_1(M_0, M)}{\Psi_{2m+2}(M_0, M)} \right|} + \text{fonction continue} \qquad (19_2)$$

* Cette condition n'est pas essentielle et peut être remplacée par la condition de l'analycité de K_m dans le domaine indiqué.

si le point M est entre les courbes droites analogues; C_m et C'_m sont des fonctions analytiques *.

§ 6. On peut tirer de l'équation (11) que $Q_z = 0$ pour $z = 0$. En effectuant dans (12) une intégration par parties, nous obtenons l'équation

$$Q(M_0) + \iint_{\sigma_0} Q_z(M)\, I_z(M_0, M)\, dS = E(M_0). \tag{20}$$

Posons

$$\Gamma(M_0, M) = \sum_{m=1}^{\infty} K_m(M_0, M) \tag{21}$$

et effectuons sur les deux membres de l'équation (20) l'opération

$$\omega(M_0) + \iint_{\sigma_0} \Gamma(M_0, M)\, \omega_z(M)\, dS. \tag{22}$$

Nous obtiendrons ainsi une équation de la forme

$$Q(M_0) + \iint_{\sigma_0} Q_z(M)\, \Gamma'(M_0, M)\, dS = F(M). \tag{23}$$

On peut choisir les fonctions $A_m(M_0, M)$ et $A'_m(M_0, M)$ de façon que la fonction Γ' soit continue et que Γ'_z soit absolument intégrable. Puisque $\Gamma'(M_0, M) = 0$ sur les $M_0\alpha_1$-courbes et $Q_z = 0$ sur l'axe u, une intégration par parties nous fournit, pour la fonction inconnue $Q(M)$, une équation intégrable de Volterra

$$Q(M_0) - \iint_{\sigma_0} Q(M)\, \Gamma'_z(M_0, M)\, dS + \int_h Q(M)\, \Gamma(M_0, M)\, du = F(M_0) \tag{24}$$

qui est équivalente à l'équation (11); h désigne l'ensemble des côtés latéraux du triangle formé par l'axe u et les tangentes aux $M_0\alpha_1$-courbes dans le point M_0. En résolvant l'équation (24) nous aurons, par cela même, résolu notre problème aux limites.

Je tiens à exprimer ma gratitude à S. L. Soboleff, qui a bien voulu me faire quelques observations utiles au cours de ce travail.

Institut Seismologique de l'Académie des Sciences de l'URSS.

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* Il n'est pas difficile de trouver une loi générale de la composition des noyaux de la forme K_m et K_n, mais nous n'en aurons pas besoin.

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MATHÉMATIQUES

UNE REMARQUE CONCERNANT LE PROBLÈME DE DIRICHLET

Par D. J. SCHERMAN

(Présenté par S. L. Sobolеff, de l'Académie, le 28.VIII.1940)

Supposons que dans l'espace xyz on considère un certain volume V borné par $m+1$ surfaces fermées $S_j\,(j=1,\ldots,m+1)$. On sait que la solution du problème de Dirichlet pour un tel domaine ne peut pas, en général, être présentée comme un potentiel de double couche. Dans le cas général cette solution ne peut être présentée que sous la forme d'une somme de potentiels de simple et de double couche. D'ailleurs, le potentiel de simple couche est lui-même défini d'une manière assez compliquée et exige la solution préliminaire du problème de Robin (1). Nous montrerons dans la suite comment on peut éviter cette dernière difficulté et obtenir ainsi d'une manière plus simple la solution du problème de Dirichlet.

Convenons, pour fixer les idées, de supposer le volume V fini. La surface totale, bornant le volume V et égale à $\sum_1^{m+1} S_j$, sera désignée par S, et on supposera que S_{m+1} est la surface extérieure qui contient à son intérieur toutes les autres surfaces $S_j\,(j=1,\ldots,m)$. Enfin les volumes (extérieurs à V) bornés par les surfaces S_j seront désignés par $V_j\ (j=1,\ldots,m+1)$.

La fonction cherchée $u(x,y,z)$ harmonique dans V sera cherchée dans la forme:

$$u(x,y,z)=\int_S \nu\,\frac{d\frac{1}{r}}{dn}\,dS+\sum_1^m \frac{1}{r_j}\int_{S_j}\nu\,dS \qquad (1)$$

où

$$r=\sqrt{(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2}$$
$$r_j=\sqrt{(x-x_j)^2+(y-y_j)^2+(z-z_j)^2}$$

$M(\xi,\eta,\zeta)$ étant un point de la surface S, $M(x_j,y_j,z_j)$ — un point arbitrairement fixé situé à l'intérieur de $V_j\,(j=1,\ldots,m)$; la normale n est supposée dirigée de l'intérieur de V à l'extérieur.

Passons à la limite dans l'égalité (1) en faisant tendre le point $M(x,y,z)$ vers le point arbitraire $M(\xi_0,\eta_0,\zeta_0)$ de la surface S. Alors pour les valeurs de $u(\xi_0,\eta_0,\zeta_0)$ données on obtient pour la détermination de la densité ν une équation intégrale de Fredholm. Il est facile de dé-

montrer que cette équation est résoluble. En effet, en considérant l'équation homogène correspondante nous aurons partout dans l'espace V:

$$\int_S \nu \frac{d\frac{1}{r}}{dn} dS + \sum_1^m \frac{1}{r_j} \int_{S_j} \nu \, dS = 0. \qquad (2)$$

Le premier membre de cette dernière égalité sera désigné par $u^*(x, y, z)$. En considérant la fonction $u^*(x, y, z)$ dans les espaces V et V_{m+1}, nous trouvons sans peine que $\frac{du^*}{dn}$ reste continue quand on passe de V à V_{m+1} par S_{m+1}. Donc, en tenant compte de ce que $u^*(x, y, z)$ s'annulle à l'infini, nous aurons $u^*(x, y, z) = 0$ partout dans V_{m+1}. En comparant avec (2) on trouve $\nu = 0$ sur S_{m+1}. Il suit alors de (2) que

$$\sum_1^m \left\{ \int_{S_j} \nu \frac{d\frac{1}{r}}{dn} dS + \frac{1}{r_j} \int_{S_j} \nu \, dS \right\} = 0$$

ce qui peut encore être écrit de la manière suivante:

$$\int_{S_e} \nu \frac{d\frac{1}{r}}{dn} dS + \frac{1}{r_e} \int_{S_e} \nu \, dS = -\sum_1^{m\prime} \left\{ \int_{S_j} \nu \frac{d\frac{1}{r}}{dn} dS + \frac{1}{r_j} \int_{S_j} \nu \, dS \right\} \qquad (3)$$

où e est un nombre quelconque fixé dans la suite finie: $1, \ldots, m$; le signe $\sum'$ montre qu'en faisant la sommation on a omis le terme correspondant à $j = e$. Introduisons à l'extérieur de S_j la fonction

$$u_j(x, y, z) = \int_{S_j} \nu \frac{d\frac{1}{r}}{dn} dS + \frac{1}{r_j} \int_{S_j} \nu \, dS \quad (j = 1, \ldots, m). \qquad (4)$$

La première partie de l'égalité (3) est une fonction harmonique hors de S_e; au contraire, la seconde partie de cette égalité est une fonction harmonique à l'intérieur de S_e. Donc, la fonction $u_e(x, y, z)$ sera harmonique dans tout l'espace et, par conséquent (comme elle s'annulle à l'infini) elle est partout identiquement nulle. Ainsi nous avons maintenant

$$\int_{S_j} \nu \frac{d\frac{1}{r}}{dn} dS + \frac{1}{r_j} \int_{S_j} \nu \, dS = 0 \text{ à l'extérieur de } S_j \ (j = 1, \ldots, m). \qquad (5)$$

En considérant cette dernière égalité à l'infini, on trouve sans peine qu'elle se décompose en égalités suivantes:

$$\int_{S_j} \nu \frac{d\frac{1}{r}}{dn} dS = 0, \quad \int_{S_j} \nu \, dS = 0 \ (j = 1, \ldots, m). \qquad (6)$$

Il en résulte aussi facilement que ν est encore nulle sur les surfaces restantes $S_j (j = 1, \ldots, m)$.

Ainsi l'équation homogène ne possède que la solution triviale, donc l'équation non homogène est résoluble par rapport à ν d'une manière unique. Après avoir déterminé ν, nous trouvons d'après la formule (2) la fonction cherchée.

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MATHÉMATIQUES

SUR LES SOUS-ANNEAUX FERMÉS DE L'ANNEAU DES FONCTIONS À n DÉRIVÉES CONTINUES

Par J. KHOURGUINE et N. TSCHETININE

(Présenté par A. N. Kolmogoroff, de l'Académie, le 5.VII.1940)

Stone (1) et Šilov (2) en considérant l'anneau topologique de toutes les fonctions réelles et continues (C), ont donné une condition nécessaire et suffisante pour qu'un sous-anneau fermé, appartenant à cet anneau, coïncide avec l'anneau entier. Dans la présente Note nous démontrerons un théorème analogue pour l'anneau topologique de toutes les fonctions réelles continues à n dérivées continues $(D_n;\ n=1,\ 2,\ \ldots)$ définies sur le segment $[0,1]$. La convergence dans D_n est définie comme la convergence uniforme des fonctions et de leurs n dérivées respectivement.

Nous allons considérer les sous-anneaux R_n de l'anneau D_n jouissant des propriétés suivantes:

1. $x(t)\equiv 1 \subset R_n$.
2. Si $x(t)\subset R_n$, alors $\lambda x(t)\subset R_n$ quel que soit λ réel.
3. R_n est fermé, c'est-à-dire la limite d'une suite de fonctions appartenant à R_n est elle-même une fonction de R_n, si cette suite converge au sens de la convergence dans D_n.

Lemme I. *Si* $x(t)\subset R_n$, *alors* $|x(t)|^{2n+1}\subset R_n$.

Sans restreindre la généralité on peut supposer que $|x(t)|<1$. Ecrivons les égalités évidentes:

$$|x(t)|^{2n+1}=x^{2n}|x|=x^{2n}\sqrt{1-(1-x^2)}=x^{2n}\sum_{m=0}^{\infty}a_m(1-x^2)^m$$

où a_m sont les coefficients de la formule du binôme.

Si l'on différentie n fois l'expression qui se trouve au second membre, on obtient, comme il est facile de vérifier, une série qui converge uniformément et présente, donc, la n-ième dérivée du premier membre. Il suit de la condition 3 que $|x(t)|^{2n+1}\subset R_n$.

Supposons que R_n jouit de la propriété de séparabilité: pour chaque couple de points du segment $[0,1]$ on peut indiquer une fonction appartenant à R_n et qui possède en ces points des valeurs différentes. On a alors le

Lemme II. *Quel que soit le point* $\xi\subset[0,1]$, *il existe une fonction* $x_\xi(t)\subset R_n$ *telle que* $x_\xi(\xi)<x_\xi(t)$ *pour tous les* $t\neq\xi$.

Pour chaque point $\eta\subset[0,1]$ qui diffère de ξ il existe d'après la propriété de séparabilité une fonction $y(t)\subset R_n$ telle que $y(\xi)\neq y(\eta)$. Alors la fonction

$$y_\eta(t)=(y(t)-y(\xi))^2$$

jouit des propriétés suivantes:

$$y_\eta(t) \geqslant 0,\ y_\eta(\xi) = 0,\ y_\eta(\eta) > 0.$$

Il suit de la continuité de $y_\eta(t)$ qu'il existe un intervalle contenant le point η et où $y_\eta(t) > 0$. En vertu du théorème de Borel-Lebesgue, de ce système d'intervalles couvrant le segment [0,1] sauf le point ξ on peut extraire un système dénombrable d'intervalles et une suite correspondante de fonctions

$$y_{\eta_1}(t),\ y_{\eta_2}(t),\ \ldots,\ y_{\eta_k}(t),\ \ldots$$

telles que pour chaque point $\eta \neq \xi$ il existe une certaine fonction $y_{\eta_k}(t)$ telle que $y_{\eta_k}(\eta) > 0$. Maintenant la fonction cherchée $x_\xi(t)$ peut être présentée comme la somme de la série

$$x_\xi(t) = \sum_{k=1}^{\infty} \lambda_k y_{\eta_k}(t)$$

où les nombres réels λ_k sont choisis de telle manière que la série converge au sens de la convergence dans D_n.

Lemme III. *Pour chaque couple de points* $\eta, \xi \subset [0,1]$ *on peut indiquer une fonction* $x_{\eta,\xi}(t) \subset R_n$ *telle que*

$$x_{\eta\xi}(t) \begin{cases} = 0 \text{ sur } [0, \eta] \\ \text{est contenue entre } 0 \text{ et } 1 \text{ sur } (\eta, \xi) \\ = 1 \text{ sur } [\xi, 1] \end{cases}$$

Pour fixer les idées nous allons supposer que $\eta < \xi$. Considérons la fonction $x_\xi(t) \subset R_n$ définie dans le lemme II et posons

$$m = \min_{0 \leqslant t \leqslant \eta} x_\xi(t).$$

Il est évident que $m > 0$. Considérons une fonction auxiliaire

$$y_\eta(t) = |x_\xi(t) - m|^{2n+1} - (x_\xi(t) - m)^{2n+1}$$

qui jouit des propriétés

$$y_\eta(t) \begin{cases} = 0 \text{ sur } [0, \eta] \\ > 0 \text{ dans un certain intervalle contenant le point } \xi \\ \geqslant 0 \text{ sur } [0,1] \end{cases}$$

Fixons η et faisons ξ parcourir tout le segment $[\xi, 1]$. On obtient un recouvrement du segment $[\xi, 1]$; en vertu du lemme de Heine-Borel on peut extraire un système fini de ces intervalles recouvrant tout le segment $[\xi, 1]$ et former une suite finie de fonctions correspondantes

$$y_{\eta_1}(t),\ y_{\eta_2}(t),\ \ldots,\ y_{\eta_k}(t)$$

définissant la fonction

$$y(t) = \sum_{m=1}^{k} y_{\eta_m}(t)$$

qui est positive sur le segment $[\xi, 1]$.

Maintenant la fonction cherchée $x_{\eta\xi}(t)$ sera présentée comme

$$x_{\eta\xi}(t) = \frac{1}{2M} \{(y(t) - M)^{2n+1} - |y(t) - M|^{2n+1} + 2M\}$$

où

$$M = \min_{\xi \leqslant t \leqslant 1} y(t).$$

Théorème 1. *Pour que le sous-anneau R_n coïncide avec tout l'anneau D_n il est nécessaire et suffisant que R_n jouit des propriétés:*

1. *de la séparabilité;*

2. *pour chaque point $\xi \subset [0,1]$ on peut indiquer une fonction $x(t) \subset R_n$ telle que $x'(\xi) \neq 0$.*

Nécessité. Si R_n coïncide avec D_n, alors la fonction $x(t) \equiv t$, par exemple, qui prend des valeurs différentes aux points différents du segment $[0,1]$, possède partout une dérivée positive et appartient à R_n.

Suffisance. a) Montrons qu'il existe dans R_n une fonction dont la dérivée est partout positive. Il suit de la propriété 2) et de la continuité de $x'(t)$ qu'il existe un $\delta > 0$ tel que $x'(t) > 0$ dans l'intervalle $(\xi - 2\delta,\ \xi + 2\delta)$. Considérons la fonction

$$x_1(t) = |x(t) - x(\xi - \delta)|^{2n+1} + (x(t) - x(\xi - \delta))^{2n+1}.$$

Dans l'intervalle $(\xi - 2\delta,\ \xi - \delta)$ on a $x_1(t) \equiv 0$. Définissons la fonction $x_2(t) = x_1(t)\, x_{(\xi-2\delta),(\xi-\delta)}(t)$, où $x_{(\xi-2\delta),(\xi-\delta)}(t)$ est la fonction construite dans le lemme III. On a $x_2(t) \equiv 0$ sur le segment $[0,\ \xi - \delta]$ et $x_2'(t) > 0$ dans l'intervalle $(\xi - \delta,\ \xi + \delta)$.

D'une manière analogue «coupons» la fonction $x_2(t)$ à droite de l'intervalle $(\xi - \delta, \xi + \delta)$. A cet effet définissons la fonction

$$x_3(t) = (x_2(t) - x_2(\xi - \delta))^{2n+1} - |x_2(t) - x_2(\xi + \delta)|^{2n+1}.$$

Dans l'intervalle $(\xi + \delta,\ \xi + 2\delta)$ on a $x_3(t) \equiv 0$. La dérivée $y_\xi'(t)$ de la fonction

$$y_\xi(t) = x_3(t)\,(1 - x_{(\xi+\delta)\,(\xi+2\delta)}(t))$$

est nulle partout sauf dans l'intervalle $(\xi - \delta,\ \xi + \delta)$, où elle est positve.

En vertu du lemme de Heine-Borel on peut extraire un système fini d'intervalles $(\xi - \delta,\ \xi + \delta)$ et former une suite finie de fonctions correspondantes $y_{\xi_1}(t), \ldots, y_{\xi_n}(t)$ dont la somme $y(t) = \sum_{k=1}^{n} y_{\xi k}(t)$ possède partout une dérivée positive.

Remarque. Il est facile de voir que si $x(t) \subset R_n$ et $F(x)$ est une fonction continue, ainsi que ses n premières dérivées, définie sur l'ensemble des valeurs de la fonction $x(t)$, l'on a $F[x(t)] \subset R_n$.

b) Soit $z(t)$ une fonction quelconque de D_n. Montrons que $z(t) \in R_n$ ce qui démontrera notre théorème.

Désignons par $y^{-1}(t)$ la fonction inverse à la fonction $y(t)$, précédemment définie. La fonction $y^{-1}(t)$ est continue ainsi que ses n premières dérivées et déterminée sur l'ensemble des valeurs de $y(t)$. En vertu de la remarque on a $y^{-1}[y(t)] \subset t \subset R_n$. D'après le théorème de Weierstrass $z^n(t)$ peut être approché uniformément au moyen de polynômes en τ, d'où (en vertu de ce que R_n est fermé) il résulte que $z(t) \in R_n$.

Les théorèmes suivants obtenus par Stone (1) et Šilov (2) peuvent facilement être démontrés de la même manière que le théorème 1.

Théorème 2*. *Pour que le sous-anneau R de l'anneau C coïncide avec l'anneau C il est nécessaire et suffisant que la propriété de séparabilité soit remplie dans R* (R est soumis aux mêmes conditions 1, 2, 3 qui ont été imposées à R_n; la convergence dans C est définie comme convergence uniforme).

La nécessité est évidente.

* Dans la démonstration du théorème se trouve contenue comme cas particulier la démonstration donnée par M. Lebesgue du théorème de Weierstrass sur l'approximation des fonctions continues au moyen de polynômes.

Suffisance. Soit $z(t)$ une fonction continue arbitraire. Considérons une division du segment (0,1] en intervalles (ξ_i, ξ_{i+1}) de continuité uniforme de $z(t)$ pour un $\varepsilon > 0$. Il est facile de voir que

$$|z(t) - \{z(0) + \sum [z(\xi_{i+1}) - z(\xi_i)] x_{\xi_i \xi_{i+1}}(t)\}| < \varepsilon$$

où $x_{\xi_i \xi_{i+1}}$ est la fonction définie dans le lemme III.

L'expression entre parenthèses est une fonction appartenant à R; comme ε est arbitraire, $z(t)$ est approchée uniformément par une suite de telles fonctions, ce qui démontre le théorème.

Théorème 3. *Soit S un espace bicompact de Hausdorff et soit $C(S)$ l'anneau de toutes les fonctions réelles et continues définies dans l'espace S. Alors pour que le sous-anneau $R(S)$ coïncide avec l'anneau $C(S)$ il est nécessaire et suffisant que la condition de séparabilité soit remplie dans $R(S)$.*

Ce théorème peut être démontré par la même méthode que le théorème 2, si dans la démonstration des lemmes I, II, III on remplace le segment [0,1] par l'espace S. Les énoncés et les démonstrations doivent être un peu changés en tenant compte des particularités de l'espace S.

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ASTRONOMIE

SUR LE NOMBRE D'ÉTOILES DANS LA GALAXIE

Par B. FESSENKOFF, de l'Académie

1. Le nombre d'étoiles dans la Galaxie était évalué par M. Seares (¹). En faisant le dénombrement d'étoiles dans certaines aires du ciel choisies uniformément par rapport à la Voie Lactée et procédant par une forte extrapolation, il a obtenu $3 \cdot 10^{10}$. Cette évaluation ne présente maintenant qu'un intérêt historique puisque l'absorption de lumière très prononcée dans notre Galaxie n'était pas prise en considération. On peut dire d'ailleurs que l'espace accessible à nos instruments est certainement assez petit par rapport à l'étendue de la Galaxie.

On peut se former une idée plus juste sur la masse totale de la Galaxie en considérant la vitesse de la rotation galactique. En supposant que les mouvements d'étoiles suivent les trajectoires circulaires, on obtient dans l'hypothèse de l'état stationnaire que la masse du système est 10^{11} de celle du Soleil.

2. Faisons l'hypothèse usuelle que la densité stellaire (le nombre d'étoiles dans l'unité de volume) est distribuée selon les surfaces de rotation symétriques par rapport au plan galactique

$$\varphi(r, z) = k^2$$

k étant un paramètre variant d'une surface à l'autre. Dans un volume élémentaire $R^2 dR\, d\omega$ qui est observé par un observateur en S sous un angle solide $d\omega$, le nombre d'étoiles est

$$N(k)\, dv = N(k)\, R^2\, dR\, d\omega.$$

Le nombre total dans la direction considérée est

$$n d\omega = d\omega \int_0^\infty N(k)\, R^2\, dR.$$

Supposons que les surfaces d'égales densités sont les ellipsoïdes de rotation

$$\frac{r^2}{a_0^2} + \frac{z^2}{b_0^2} = k^2.$$

Alors le nombre total d'étoiles est

$$\overline{N} = 4\pi \int_0^k N(k)\, a_0^2\, b_0 k^2 dk.$$

Adoptons maintenant que la densité stellaire en fonction k est

$$\left.\begin{aligned} N(k) &= A_0 + \frac{A_1}{a_0 k_0} + \frac{A_2}{a_0^2 k_0^2} + \dots \quad \text{pour } k \leqslant k_0 \\ \text{et} \qquad & \\ N(k) &= A_0 + \frac{A_1}{a_0 k} + \frac{A_2}{a_0^2 k^2} + \dots \quad \text{pour } k \geqslant k_0 \end{aligned}\right\} \qquad (1)$$

où A_0, A_1, A_2 sont les constantes à déterminer. Dans ce cas

$$\overline{N} = \frac{4\pi}{3} r^3 \eta A_0 + 2\pi A_1 r^2 \eta \left(1 - \frac{1}{3}\frac{k_0^2}{k^2}\right) + 4\pi A_2 r \eta \left(1 - \frac{2}{3}\frac{k_0}{k}\right)$$

ou bien

$$\overline{N} = \frac{4\pi}{3} r^3 \eta A_0 + 2\pi A_1 r^2 \eta + 4\pi A_2 r \eta$$

k_0 étant très petite par rapport à k. Le point singulier dans l'origine n'a donc pas d'importance.

Introduisons une variable nouvelle ϑ suivant la relation

$$R = \frac{l \sin(\vartheta + \alpha)}{\sin \vartheta}$$

qui est tirée du triangle SCK (S — le Soleil, C — centre du système, K — le volume élémentaire dc; $SC = l$; $SK = R$; $\alpha = \angle KSC$; $\vartheta = \angle CKS$). Évidemment

$$\cos \alpha = \cos \beta \cos \lambda$$

β, λ étant les coordonnées sphériques usuelles qui fixent la direction SK par rapport au plan galactique. On a

$$n = \int_{\vartheta_0}^{\pi - \alpha} N(k) \frac{l^3 \sin^2(\vartheta + \alpha)}{\sin^4 \vartheta} \sin \alpha \, d\vartheta.$$

En tenant compte que

$$z = R \sin \beta; \quad r^2 = l^2 + R^2 \cos^2 \beta - 2lR \cos \beta \cos \lambda$$

on trouve la relation suivante entre k et ϑ:

$$a_0^2 k^2 = l^2 \left(1 + \frac{\sin^2(\alpha + \vartheta) \cos^2 \beta}{\sin^2 \vartheta} - 2 \frac{\sin(\alpha + \vartheta)}{\sin \vartheta} \cos \beta \cos \lambda + \frac{\sin^2(\alpha + \vartheta)}{\eta^2} \frac{\sin^2 \beta}{\sin^2 \vartheta}\right).$$

Considérons particulièrement la direction vers le pôle galactique ($\beta = 90°$; $\alpha = 90°$) et celle dans le plan de la Galaxie normalement au rayon du système ($\lambda = 90°$; $\beta = 0°$; $\alpha = 90°$). Dans le premier cas

$$n_1 = l^3 A_0 \int_{\vartheta_0}^{\frac{\pi}{2}} \frac{\cos^2 \vartheta}{\sin^2 \vartheta} d\vartheta + l^3 A_1 \int_{\vartheta_0}^{\frac{\pi}{2}} \frac{\cos^2 \vartheta \, d\vartheta}{\sin^4 \vartheta \sqrt{1 + \frac{1}{\eta^2} \operatorname{ctg}^2 \vartheta}} +$$

$$+ A_2 l \int_{\vartheta_0}^{\frac{\pi}{2}} \frac{\cos^2 \vartheta \, d\vartheta}{\sin^4 \vartheta \left(1 + \frac{1}{\eta^2} \operatorname{ctg}^2 \vartheta\right)}.$$

Dans le second cas on a la même expression où l'on doit poser $\eta = 1$. La limite inférieure d'intégration est donnée par la condition évidente

$$\operatorname{ctg} \vartheta_0 = \eta \frac{\sqrt{r^2 - l^2}}{l} = \eta \operatorname{ctg} \psi_0.$$

On peut poser d'après les données modernes que

$$l = 0.667\, r$$

r étant le rayon galactique. En faisant l'intégration, on trouve

$$n_1 = l^3 A_0 \eta^3 \frac{\operatorname{ctg}^3 \psi_0}{3} + A_1 l^2 \eta^3 \left(\frac{1}{4 \log e} \cdot \log \frac{1 - \cos \psi_0}{1 + \cos \psi_0} + \frac{1}{2} \frac{\cos \psi_0}{\sin^2 \psi_0} \right) + \\ + A_2 l \eta^3 \left(\operatorname{ctg} \psi_0 - \frac{\pi}{2} + \psi_0 \right). \qquad (2)$$

Il en résulte que

$$n_1 = n_2 \eta^3$$

c'est-à-dire

$$\frac{n_2}{n_1} = C = \frac{a^3}{b^3}.$$

Nous avons donc le théorème suivant.

La concentration galactique C mesurée comme le rapport du nombre d'étoiles n_2 et n_1 est égale au cube du rapport des demi-axes de l'ellipsoïde représentant la surface d'égale densité stellaire. On peut dire autrement que la concentration galactique C ne dépend que de la forme de la Galaxie.

On voit donc qu'il est possible de déterminer le nombre total d'étoiles dans la Galaxie n'évaluant que leur nombre dans une direction déterminée, la concentration C et le caractère de la fonction $N(k)$ étant supposés connus. Cette évaluation est possible pour le pôle galactique. En effet, le nombre d'étoiles observées dans cette direction croît de moins en moins rapidement avec la magnitude. Ceci permet de faire l'extrapolation assez sûre de leur nombre total.

Les données statistiques de Seares mentionnées plus haut conduisent ainsi à la conclusion que le nombre total d'étoiles par degré carré dans la direction du pôle galactique est égal à 10^4. Donc

$$n_1 = \left(\frac{180}{\pi} \right)^2 \cdot 10^4$$

$$\overline{N} = \frac{n_1}{r^2} \times$$

$$\times \frac{\frac{4\pi}{3} r^3 A_0 + 2\pi A_1 r^2 + 4\pi A_2 r}{\frac{\operatorname{ctg} \psi_0}{3} l^3 A_0 + \left(\frac{1}{4 \log l} \log \frac{1 - \cos \psi_0}{1 + \cos \psi_0} + \frac{1}{2} \frac{\cos \psi_0}{\sin^2 \psi_0} \right) A_1 l^2 + A_2 l \left(\operatorname{ctg} \psi_0 - \frac{\pi}{2} + \psi_0 \right)}$$

3. La distribution d'étoiles en fonction de la distance du centre galactique n'est pas connue. On peut pourtant obtenir une certaine idée sur cette distribution en comparant la Voie Lactée à d'autres formations analogues — les nébuleuses extragalactiques. J'ai montré que la brillance de la nébuleuse d'Andromède qui ressemble extrêmement à la Voie Lactée varie d'une façon rapide et continue du centre à la périphérie. Cette variation peut être représentée par la formule

$$f(r) = a + \frac{b}{r} + \frac{c}{r^2}, \quad a = -0.04; \quad b = 0.11; \quad c \approx 0$$

l'intervalle angulaire de 40′ étant pris pour unité. On voit donc que la variation de la brillance est plus rapide que r^{-1}, mais un peu plus lente que r^{-2}.

Il est à croire que la même formule pourrait être adoptée pour la distribution de la densité stellaire dans le plan de la symétrie, étant donné que les nébuleuses extragalactiques possèdent la structure très plate. J'ai montré pourtant que l'indice de couleur diminue aussi avec la distance du centre. La température de couleur augmente de 6000° au centre jusqu'a 10000° à la

distance de 40′. Il en résulte que les parties périphériques de la nébuleuse d'Andromède sont plus riches en étoiles de température élevée, dont le rendement d'énergie par l'unité de masse est plus considérable. Par conséquent, la variation de la densité stellaire en fonction de r doit être plus rapide que la variation de la brillance visuelle, en approchant peut etre à r^{-2}.

Adoptons donc que $A_0 = A_1 = 0$. Alors

$$\overline{\underline{N}} = \frac{n_1}{\eta^2} \frac{4\pi r}{l\left(\operatorname{ctg}\psi_0 - \frac{\pi}{2} + \psi_0\right)}$$

ou bien

$$\overline{\underline{N}} = 2.21\,\eta^{-2} \cdot 10^9.$$

Si, d'autre part, la densité stellaire dans le plan galactique varie comme r^{-1}, on a $A_0 = A_2 = 0$ et

$$\overline{\underline{N}} = \frac{n_1}{\eta^2} \frac{2\pi r^2}{l^2\left(\frac{1}{4\log e}\log\frac{1-\cos\psi_0}{1+\cos\psi_0} + \frac{1}{2}\frac{\cos\psi_0}{\sin^2\psi_0}\right)}$$

d'où

$$\overline{\underline{N}} = 1.3\,\eta^{-2} \cdot 10^9. \qquad (3)$$

On voit que la valeur N dépend plutôt de la forme de la Galaxie, c'est-à-dire de la valeur η qui est intimement liée avec la concentration galactique C.

On sait que C observée directement augmente rapidement avec la magnitude. En comptant le nombre d'étoiles jusqu'à la 21-ième gr., on obtient $C = 44.2$. La valeur réelle de C est sans doute beaucoup plus considérable. En tenant compte de l'absorption galactique, on trouve que l'espace accessible à l'observation est bien plus petit dans la direction vers le plan galactique que dans la direction vers le pôle (pour les étoiles de 21-ième gr. les distances correspondantes sont 2700 psc et 7600 psc). Si l'on compte les étoiles dans les deux directions jusqu'aux mêmes distances du Soleil, on trouve la valeur C égale à 2200. En réalité, la valeur réelle de C est égale au rapport du nombre d'étoiles comptées entre les limites des surfaces de densité égale qui possèdent un fort aplatissement. Il est évident que la valeur C ainsi évaluée doit être énorme. J'ai obtenu approximativement que $C = 10^5 - 10^6$.

En posant, par conséquent, $\eta = 10^{-2} - 2 \cdot 10^{-2}$, on tire de (3) que $\overline{\underline{N}} \approx 10^{12}$.

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ASTRONOMY

ON THE POSSIBLE MECHANISM OF EJECTION OF MATTER BY NOVAE AT THE MOMENT OF THEIR MAXIMUM BRIGHTNESS

By E. R. MUSTEL

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It has been ascertained on the ground of experimental investigation that the course of development of a typical Nova to the moment of its maximum brightness is characterized by a continuous increase in brightness and by a relatively stable absorption spectrum with absorption lines exhibiting Doppler displacement in the direction of short wave-lengths. More or less clearly expressed emission components are in this case observable on the red edge of the displaced absorption lines.

It is now usually believed that both the aforesaid phenomenon and some facts of minor importance are caused by a continuous spherically-symmetrical expansion of the extended envelope surrounding a Nova. Towards the moment of maximum the extension of such an envelope becomes very great. In particular, gravity acceleration in the upper layers of such an envelope, as established by various investigations, is liable to fall 10^4—10^5 times. Almost immediately after maximum brightness, there occur in the spectra of Novae changes which are quite exceptional in respect of the rapidity of their advent. The above-mentioned emission components became suddenly developed into broad bands, on the violet edges of each of which one or several new absorption lines make their appearance. The thus formed new absorption systems frequently show by far greater Doppler displacements than is the case with that single absorption system which exists previous to the maximum. As a typical example of the just stated changes we may point to the outburst of N Pict 1925. In the course of no less than 15 days preceding the maximum, the single system of absorption lines corresponding to spectrum cF5 exhibited a practically constant Doppler displacement of the order of 65 km/sec. At the same time immediately after the maximum, the development of the spectrum brought about in one or two days the formation of new absorption systems with velocities of 100, 310 and 1100 km/sec. All changes similar to the one just referred to can at present be successfully interpreted as a powerful ejection of matter by a Nova at the moment of its maximum brightness. The only conceivable reason for such an ejection lies, to all appearances, in radiative pressure. The growing significance of the latter with the approach to the maximum is not due in this case to the direct rise of the brightness of the star, as a whole, but depends on the continuous slowing down of gravity acceleration in the upper layers of the expanding envelope. However, such a simple mechanism, in the case of which at a certain moment radiation pressure on the envelope exceeds gravitation, wherefore acceleration of envelope becomes positive, can, by no means, account for those velocities which arise within an exceedingly short lapse of time (from several hours to 2 or 3 days).

The theory of the ejection of high-speed atoms from the Sun and stars developed by Milne (1) in 1926 is also unable to afford an explanation to the above-described phenomenon. This theory, as is well known, is in its pure form applicable solely to the emission of individual atoms. At the same time such a powerful and short-time ejection of matter as is observable with Novae is, from the angle of Milne's theory, altogether unaccountable. Besides this, the fact also remains obscure that all these phenomena arise precisely after the maximum, while the envelope of any new star is sufficiently extensive also before the maximum. At the same time the suggestion of Mc Crea (2) to the effect that in the expanding envelope surrounding a Nova a certain velocity gradient which is positive in the direction from the centre of the star should necessarily be available, and that, consequently, Milne's mechanism may safely be applied, seems in a certain measure to hold true. However here again it remains unclear why this mechanism should be active only after the maximum, since, for instance, as shown by A. Beer (3) for N Her 1934, the difference in the velocities of expansion of the photosphere and of the outer layers determining the presence of absorption lines existed during the whole period of the increase of the star's brightness up to the maximum (during 9 days). And yet ejection of matter began only after the maximum.

On the other hand, calculations performed by Mc Crea (4) have shown that very close to the maximum of brightness, radiation pressure in the upper layers of the expanding envelope becomes a factor of exceptional importance. Applying the obtained results to Milne's theory, Mc Crea found that the atoms of hydrogen and ionized calcium when entering a field radiation, characterized only by a continuous spectrum, are capable of acquiring velocities even greater than the observed ones. Thus, if we are to believe that it is radiation pressure which is the actual cause for the ejection of envelopes at the time of maximum, we should be able to explain: 1) why such ejection occurs, and why this takes place immediately after the maximum; 2) why several absorption systems are observable.

A possible interpretation of the first of these phenomena is given below. As regards the second, the author intends to discuss it in the nearest future.

Let us consider the outer layers of an expanding Nova shortly before the moment of its maximum. The motion of these layers will be governed by the following forces: 1) radiation pressure, 2) forces connected with the gas gradient, 3) gravitational attraction by the principal mass of the star. To give a quantitative relationship between these forces appears at present impossible on account of lack of respective theoretical investigations. Nevertheless, a number of considerations seem to show that radiation pressure can hardly play an important part in the present case. In respect of spectral lines, this contention will be sufficiently obvious, since in the spectrum regions held by them, the radiation stream passing through the upper layers of the envelope under consideration is greatly weakened*. As to the radiation pressure caused by continuous absorption beyond the heads of the spectrum series, it appears here sufficient to consider hydrogen alone, since for the usual temperatures characterizing Novae at the maximum, continuous absorption by metals will be small by virtue of their relatively low content. For the same temperatures the part played by the Lyman continuum is, as may easily be shown, small, too. Thus, there remains only continuous absorption by hydrogen atoms for frequencies that are smaller than the frequencies of the Lyman continuum. Yet, immediately after the maximum the situation changes radically. In order to under-

* In such a measure in which decrease of radiation in absorption lines can be recorded by observations.

stand this, let us consider those processes which take place in the envelopes of Novae during maximum brightness. Spectrophotometrical investigations have brought to light the fact that, close to the moment of the maximum, the temperature of a typical Nova remains practically constant. Accordingly, the existence of a maximum testifies by itself only to the circumstance that the star, having reached its utmost size, begins to contract rapidly by dint of the weakening of the previously acting energy sources, this contraction being, in the main, effected at the expense of the reduction of the effective radius of its «photosphere». At the same time, the outermost layers of the envelope of the expanded star must necessarily become detached from the latter. In point of fact, the only forces pulling them in the direction of the centre of the Nova will be gravitational forces. However, from computations accomplished by the author for typical new stars it can be seen that for the external layers of an envelope surrounding such stars the following inequality will in all cases hold true:

$$V_{r_{\max}} \geqslant \frac{2k^2M}{r_{\max}} \qquad (1)$$

where $V_{r\,\max}$ is expansion velocity of envelope at moment of maximum* found on the basis of absorption line displacement (the first absorption-system); $r_{\max}$ is the radius of the envelope at the same moment taken on the ground of various investigations similar to those of Mc Laughlin ([5]). For M — the mass of Novae—we adopted a magnitude equal to 10 M $\odot$. A certain exception is afforded by N Pict. 1925 where inequality (1) is fulfilled at a mass not exceeding 5 M $\odot$. However, in the present case the magnitude assumed for $r_{\max}$ is connected in the main with «the photosphere» of the star in its maximum. At the same time Beer ([3]) showed for N Her 1934 that the radius determining the upper layers of the «reversing layer» of a Nova is by far larger than the radius of its «photosphere». There is no doubt whatever that the same will hold true also with respect of N Pict. 1925. And if this is indeed the case, the terminal magnitude for mass to enter expression (1) should in this instance, too, be taken as equal to 10 M $\odot$.

Thus, even in the case when the forces connected with radiation pressure at the moment of maximum happen to be negligibly small, yet the inertia possibilities of the upper layers of the envelope of a typical Nova will play the decisive part, and these layers will become severed from the bulk of the star. At the same time the more innerly located parts of the original envelope which will remain with the star, will, upon compression of the latter, follow it, forming a certain extended and relatively static atmosphere. At the moments immediately following the maximum this atmosphere will be characterized by absorption lines which will be practically undisplaced or else displaced in the direction of the red. However, inasmuch as the detached envelope will have a certain (and a rather high) velocity relatively to the star, it is self-evident that the atoms of this envelope will, owing to Doppler's effect, absorb the radiation now emitted only by the continuous spectrum connected with extended atmosphere. Accordingly, no sooner will the above stated process of detachment take place than at once after the maximum in the regions occupied by spectral lines a powerful stream of «continuous radiation» will begin falling on the inner surface of the detached envelope. In result of this within a very short period the envelope will acquire a high positive increment of velocity which will become slackened only when owing to the effect of the law

$$F \infty r^{-2} \qquad (2)$$

* Prior to the moment of detachment of envelope.

stream F, taken per unit area of detached envelope, will appreciably decrease. Law (2) explains perfectly the shape of radial velocity curves after the moment of maximum brightness. As regards the question as to whether the analyzed mechanism is capable of giving such velocities which would be sufficient to satisfy observational evidence, this problem is solved by the above-mentioned calculations of Mc Crea.

Finally, concerning the question of why the velocities of the ejection of atoms of different elements are nearly the same, we must say that the reason for this evidently lies in the fact that hydrogen owing to its exceptionally high content carries along with itself all the other elements. This problem was discussed by Mc Crea ([6]). In connexion with the above considerations on the suddenly rising rôle of the radiation, it appears of interest to trace the fate of that extended atmosphere which remains with the star after the process of detachment. Owing to the fact that after the maximum the compression of the star itself proceeds with a relatively low velocity, the absorption lines arising in such an atmosphere should necessarily be sufficiently sharp and the involved spectrum should recall the undisplaced spectrum of α Cyg. It is quite clear that this spectrum will be visible but under extremely favourable conditions, as the centres of the respective absorption lines forming the mentioned spectrum will coincide with the centres of broad emission bands. However, F. Stratton ([7]) (in his works about N Per 1901 and N Gem 1912) points out that in both stars such a non-displaced spectrum of the α Cyg. type actually existed, the truth of this assertion being, according to him, entirely undisputable. The same can be remarked concerning N Agl 1918. Thus, these experimental facts find themselves a rational explanation.

In conclusion it is necessary to set forth the following observational fact. As follows from the above-stated, it is evident that the most important spectral region for all the considered effects associated with radiation pressure is the ultra-violet region of the spectrum with frequencies greater than the frequency of Balmer's series limit. At the same time spectrophotometrical investigations show that, in N Gem 1912 and N Her 1934 ([3]), a strong continuous emission spectrum was observable even at the maximum beyond the limit of this series*.

It is here obvious that this spectrum must be connected not only with the upper parts of the extended fluorescing envelope, but also with the photosphere of a new star, as, for instance, in the case of N Her 1934, the effective temperature of the star near the maximum was clearly too small to cause a powerful fluorescence at the expense of only the Lyman continuum associated with the photosphere**. Thus, this experimental fact convinces us still more of the predominant part played by radiation pressure in all processes of ejection of matter by Novae after the maximum of their brightness.

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* In relation to other Novae no similar investigations were conducted.

** The author believes, as a most plausible presumption, that the mentioned spectrum is a recombinational spectrum connected in the main with the ionization of hydrogen atoms by means of collisions.

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THERMODYNAMIK

ÜBER DIE WÄRMEABGABEZAHL IN EINEM ZYLINDRISCHEN ROHR

Von P. W. TSCHERPAKOW

(Vorgelegt von N. E. Kotchin, Mitglied der Akademie, den 2. VIII.1940)

Wie bekannt, lässt sich das Temperaturfeld in einem sich bewegenden Medium bei einem stationären Haushalt durch die Lösung der Gleichung

$$\operatorname{div}(\lambda \operatorname{grad} T) - c_p \rho (w \operatorname{grad} T) \tag{1}$$

schildern; hier sind: T — die Temperatur des Stromes, w — der Geschwindigkeitsvektor, c_p — die Wärmekapazität, ρ — die Dichte und λ — die Wärmeleitungszahl, die eine konstante Grösse oder auch eine Koordinatenfunktion sein kann.

Die Gleichung (1) für den Fall eines Rohres mit kreisförmigem Querschnitt vom Halbmesser r_0 kann nach Umwandlung in zylindrische Koordinaten (r, φ, z) in folgender Form dargestellt werden:

$$\frac{\partial}{\partial r}\left(\lambda r \frac{\partial T}{\partial r}\right) = c_p \rho r u \frac{\partial T}{\partial z} \tag{2}$$

worin u — die Komponente des Vektors, w — eine konstante Grösse oder eine Funktion von r ist.

Da hier eine axiale Symmetrie vorhanden ist und die in der Richtung der Bewegungsgeschwindigkeit geleitete Wärme nur einen geringen Einfluss ausüben kann, setzen wir:

$$\frac{\partial}{\partial \varphi}\left(\frac{\lambda}{r}\frac{\partial T}{\partial \varphi}\right) = 0 \quad \text{und} \quad \frac{\partial}{\partial z}\left(\lambda r \frac{\partial T}{\partial z}\right) = 0. \tag{α}$$

Es seien für die Funktion $T(r, z)$ gleichartige Grenzbedingungen vorausgesetzt. Mit $T = e^{-\omega z} f(r)$ erhält man aus Gleichung (2) eine Sturm-Liouville'sche Gleichung

$$\frac{d}{dr}\left(\lambda r \frac{df}{dr}\right) + \omega c_p \rho r u f = 0. \tag{3}$$

In dieser Gleichung ist $f(r)$ eine unbekannte Funktion und ω — ein undefinierter Parameter. Die Grenzbedingungen werden auch für $f(r)$ gleichartig sein. Eine Gleichung solcher Art wurde eingehend untersucht [1] und es wurde die Existenz eines Spektrums von charakteristischen Zahlen bewiesen, von denen die erste auch festgestellt wurde. Ausserdem sind für die Gleichung (3) diese Zahlen positiv und können nach zunehmender Grösse angeordnet werden

$$\omega_0 < \omega_1 < \omega_2 < \ldots < \omega_n < \ldots \tag{β}$$

Die Lösungen von Gleichung (3) bilden ein System von Fundamentalfunktionen, die wir durch $f_0, f_1, \ldots, f_n$ bezeichnen. Dann wird das Temperaturfeld durch die Lösung der Gleichung (2) in folgender Gestalt bestimmt:

$$T(r, z) = \sum_{n=0}^{\infty} a_n e^{-\omega_n z} f_n(r) \tag{4}$$

worin die Beiwerte a_n in Abhängigkeit von den Bedingungen $T = F(r)$, d. h. der Temperatur im Anfangsbereich des Rohres, stehen.

Die Wärmeabgabezahl wird allgemein nach Formel

$$\alpha = \frac{q_0}{T} \tag{5}$$

berechnet, worin q_0 — der spezifische Wärmestrom in der zu der Rohrwandung normalen Richtung und $\overline{T}$ — die Mitteltemperatur im Schnitte z sind. Diese Werte werden bestimmt durch

$$\overline{T} = \frac{\int_0^{r_0} Tur dr}{\int_0^{r_0} ur dr} \quad \text{und} \quad q_0 = -\left(\lambda \frac{\partial T}{\partial r}\right)_{r=r_0}. \tag{6}$$

Wenn man aber aus der Differentialgleichung (2) den Wert $\left(\frac{\partial T}{\partial r}\right)_{r=r_0}$ ermittelt, so erhält man

$$q_0 = -\frac{c_p \rho}{r_0} \int_0^{r_0} \frac{\partial T}{\partial z} ur dr. \tag{7}$$

Unter Benützung des Ausdrucks für die Mittelgeschwindigkeit $\overline{u} = \frac{2}{r_0^2} \int_0^{r_0} ur dr$, kann man, indem man die Gleichungen (5), (6) und (7) heranzieht, die Formel für Nusselts Parameter in folgender Gestalt erhalten

$$Nu = -\frac{Ped}{4} \frac{\int_0^{r_0} \frac{\partial T}{\partial z} ur dr}{\int_0^{r_0} Tur dr}. \tag{8}$$

Hier sind: $Nu = \frac{\alpha d}{\lambda}$ — der Parameter von Nusselt, $Pe = \frac{c_p \rho d \overline{u}}{\lambda}$ — derjenige von Pekle und d — der Rohrdurchmesser.

Nach Einsetzung von (4) in (8) erhält man den Wert der lokalen Wärmeabgabezahl oder den entsprechenden Wert für den Parameter von Nusselt als Funktionen des gesamten Temperaturfeldes. Wenn man nun den Mittelwert des Nusselt'schen Parameters längs des Rohres in der Gestalt von $\overline{Nu} = \frac{1}{l} \int_0^l Nu\, dz$ darstellt und die Beziehung (8) einsetzt, so gelangt man nach erfolgten Berechnungen zu

$$\overline{Nu} = \frac{Pe \cdot d}{4l} \ln \frac{T_0}{T_l} \tag{9}$$

worin T_0 und T_l die Werte der Mitteltemperatur für $z=0$ und $z=l$ sind. Ausserdem lässt sich der Wert $\overline{Nu}$ in Abhängigkeit vom Temperaturfeld (4) in folgender Weise darstellen:

$$\overline{Nu}=\frac{Pe\cdot d}{4}\left[\omega_0+\frac{1}{l}\ln\frac{a_0\bar{f}_0+a_1\bar{f}_1e^{-(\omega_1-\omega_0)l}+\ldots+a_n\bar{f}_ne^{-(\omega_n-\omega_0)l}+\ldots}{a_0\bar{f}_0+a_1\bar{f}_1+\ldots+a_n\bar{f}_n+\ldots}\right] \quad (10)$$

worin $\bar{f}_n=\int_0^{r_0} f_n(r)\,urdr$ ist und die übrigen Werte schon früher ermittelt wurden.

Der Ausdruck (10) hat einen Sinn, nur wenn die Zahlenreihe, die in dem Nenner unter dem Zeichen des Logarithmus steht, eine konvergierende ist. Diese Bedingung ist erfüllt wenn die Reihe, welche sich aus der Beziehung (4) mit $z=0$ ergibt, gleichmässig konvergiert und somit eine gliederweise Integration ermöglicht. Dann ist die gleichmässige Konvergenz der Reihe, die im Zähler steht, für einen beliebigen Wert $l\geqslant 0$ auf Grund von Abel's Satz leicht zu beweisen (2). Nunmehr ist es möglich den Grenzübergang für $l\to\infty$ in der Beziehung (10) auszuführen. Mit der Bezeichnung $\lim_{l\to\infty}\overline{Nu}=N_\infty$ erhält man für den Grenzfall:

$$N_\infty=\frac{Pe\cdot d}{4}\omega_0. \quad (11)$$

Beim Vergleichen der Beziehungen (9) und (11) wird es klar, dass $\omega_0=-\lim_{l\to\infty}\frac{\ln T_l}{l}$. Somit darf man behaupten, dass der Grenzwert der Wärmeabgabezahl und der entsprechende Wert von Nusselt's Kennzahl als Funktionen der ersten charakteristischen Zahl der Gleichung (3) erscheinen, d. h. dass sie keine Funktionen des gesamten Temperaturfeldes sind und mit $z=0$, z. B., von der Temperatur gänzlich unabhängig bleiben. Dabei kann man praktisch annehmen, dass diese Werte mit den mittleren Werten für eine entsprechend gewählte Rohrlänge kongruieren.

Wenn man Nusselt's Lösung (3) für das Temperaturfeld bei einem laminären Fliessen des Stromes benützt, worin der Wert der ersten charakteristischen Zahl $\omega_0=\frac{14.627}{Pe\cdot d}$ ist, so erhält man nach der Formel (11) $N_\infty=3.66$.

Bei der ähnlichen Ausnutzung der Lösung von Latzko (4) für den Fall eines turbulenten Stromes stellt man fest, dass $N_\infty=0.0377\,Pe\,Re^{-\frac{1}{4}}$ ist. Dieselben Werte erhält man in den behandelten Beispielen auf etwas andere Weise. Die abgeleiteten Formeln (8), (10) und (11) sind anwendbar für jede Strömungsart in einem kreisrunden Rohr unter der Bedingung der axialen Symmetrie.

Bisher wurde die Ermittlung der Wärmeabgabezahl ohne Berücksichtigung der längs der Achse geleiteten Wärme durchgeführt; dagegen ermöglicht das von uns angewandte Verfahren auch dem Einfluss dieses Faktors auf die Wärmeabgabe in Sonderfällen gerechnet zu werden. Es sei der Fall des Stromflusses von einer beständigen Geschwindigkeit näher behandelt. Dann wird die Gleichung der Wärmeübergabe zu

$$\frac{\partial^2T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2T}{\partial z^2}=\frac{v}{a}\frac{\partial T}{\partial z}. \quad (12)$$

Hier sind $a=\frac{\lambda}{c_p\rho}$ und die Geschwindigkeit v konstante Grössen.

Die Randbedingungen seien $T(r_0) = 0$, $T = 0$ für $z \to \infty$ and $T = T_0$ für $z = 0$. Durch Ermittlung der Wärmeabgabezahl nach den Formeln (6) und (7) findet man

$$\alpha = -\frac{\lambda r_0}{2} \frac{\int_0^{r_0} \left(\frac{v}{a} \frac{\partial T}{\partial z} - \frac{\partial^2 T}{\partial z^2} \right) r dr}{\int_0^{r_0} T r dr}. \tag{13}$$

Mit $\theta = \int_0^{r_0} T r dr$ und $Nu = \frac{\alpha d}{\lambda}$ erhält man aus der Beziehung (13)

$$\frac{d^2\theta}{dz^2} - \frac{v}{a} \frac{d\theta}{dz} - \frac{Nu}{r_0^2} \theta = 0. \tag{14}$$

Die den Randbedingungen genügende Lösung der Gleichung (14) lautet

$$\theta = \theta_0 e^{\left(\frac{v}{2a} - \sqrt{\frac{v^2}{4a^2} + \frac{Nu}{r_0^2}} \right) z}. \tag{15}$$

Hieraus erhält man leicht

$$Nu = \left(\frac{d}{2l} \ln \frac{\theta_0}{\theta_l} \right)^2 + \frac{Pe \cdot d}{4l} \ln \frac{\theta_0}{\theta_l}. \tag{16}$$

Durch den Vergleich des Ausdrucks (16) mit der Formel (9) kann festgelegt werden, dass die Wärmeleitung längs der Achse durch den ersten Summanden berücksichtigt wird. Für geringe Werte von l, d. h. im Anfangsbereich des Rohres, kann sich der Einfluss des ersten Gliedes auf die Wärmeübergabe auswirken. Das Temperaturfeld sei als Lösung der Gleichung (12) dargestellt:

$$T = \sum_{n=0}^{\infty} a_n e^{-\omega_n z} J_0(\mu_0 x). \tag{17}$$

Hier sind $x = \frac{r}{r_0}$, $J_0(\mu_n x)$ — Bessel's Funktion erster Art, μ_n — Nullwerte dieser Funktion, $\omega_n = \frac{v}{2a} - \sqrt{\frac{v^2}{4a^2} + \frac{\mu_n^2}{r_0^2}}$, wenn $\frac{\partial^2 T}{\partial z^2} \neq 0$, und $\omega_n = \frac{a}{v} \left(\frac{\mu_n}{r_0} \right)^2$, wenn $\frac{\partial^2 T}{\partial z^2} = 0$, und $a_n = \frac{2T_0}{\mu_n J_1(\mu_n)}$ sind.

Genaue Berechnungen führen zu einem einzigen Wert des Grenzparameters von Nusselt für verschiedene Werte von ω_n; nämlich $N_\infty = 5.76$. Folglich hat die Wärmeleitung keinen Einfluss auf den Grenzwert der Nusseltschen Kennzahl in dem betrachteten Falle des Wärmeaustausches

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THERMODYNAMIK

ZUR THEORIE DES WÄRMEAUSTAUSCHES IN EINEM TURBULENTEN STROME

Von P. W. TSCHERPAKOW

(Vorgelegt von N. E. Kotchin, Mitglied der Akademie, den 2.VIII.1940)

In der vorliegenden Arbeit ist der Versuch gemacht, den Wärmeaustausch in einem Strom vom Standpunkt der Turbulenztheorien von Prandtl (1) und Karmann (1) zu untersuchen. Diese Theorien wurden in bezug auf Flüssigskeitsbewegung in einem kreisrunden Rohr bei grossen Geschwindigkeiten und für grosse Reynolds'sche Zahlen bis zu $3 \cdot 10^6$ aufgestellt. Eine Aufgabe ähnlicher Art findet man in dem Artikel von Latzko (2), doch ist die vom letzteren erhaltene Lösung nur für den Wirkungsbereich des Widerstandsgesetzes von Blasius, d. h. für solche Reynolds'sche Zahlen, die 10^5 nicht überschreiten, anwendbar.

Für einen Strom in einem Rohre vom Halbmesser r_0 kann die Grösse der tangentiellen Beanspruchung nach der Theorie von Prandtl als

$$\tau = \rho l^2 \left(\frac{\partial u}{\partial r}\right) \tag{1}$$

dargestellt werden, worin u—die Geschwindigkeit, ρ—die Dichte, r—den veränderlichen Abstand von der Achse und l—die Länge des Mischungsweges bezeichnen.

In der Voraussetzung einer mechanischen Ähnlichkeit im turbulenten Strome nimmt Karmann annäherend $l = k \frac{u'}{u''}$ an, aber in der Nähe der Wandung kann man nach Prandtl $l = kr_1$ nehmen, wobei k— eine konstante Grösse und r_1— der veränderliche Abstand von der Wandung sind.

Die Integration der Gleichung (1) ermöglicht die Geschwindigkeitenverteilung in dem Strome festzustellen und zwar für eine entwickelte Turbulenz, da eine willkürliche Konstante in der allgemeinen Lösung in Abhängigkeit von der Bedingung bestimmt wird, dass für grosse Reynolds'sche Zahlen der Geschwindigkeitsgradient in der Nähe der Wandung sehr gross ist. Für den Fall $l = kr_1$ wird die Geschwindigkeitenverteilung zu

$$u = u_m + \frac{v_*}{k} \ln\left(1 - \frac{r}{r_0}\right). \tag{2}$$

Hier sind: u_m—die maximale Geschwindigkeit an der Achse und $v_* = \sqrt{\frac{\tau_0}{\rho}}$, worin τ_0—die tangentielle Beanspruchung an der Wandung ist.

In seinen Versuchen fand Nikuradze (1), dass Gleichung (2) gut mit der beobachteten Geschwindigkeitenverteilung übereinstimmte, nur musste

$k=0.4$ gesetzt werden. Es sei T die Temperatur; dann erhält man für einen Wärmestrom

$$q=-\lambda\frac{\partial T}{\partial r}. \tag{3}$$

Laut Newtons Hypothese wird die tangentielle Beanspruchung in der Gehalt

$$\tau=-\mu\frac{\partial u}{\partial r} \tag{4}$$

dargestellt.

Wenn man λ und μ als gewisse Funktionen von r betrachtet, so kann man nach Karmann und Latzko λ als turbulente Wärmeleitungszahl und μ als turbulente Zähigkeitszahl bezeichnen. Nach dem Prinzip der Analogie zwischen Reibung und Wärmeübergabe, das noch von Reynolds vorgebracht wurde, wird zwischen diesen Werten das Vorhandensein eines Zusammenhangs in der Gestalt von $\frac{\mu c_p}{\lambda}=1$ vorausgesetzt, worin c_p die Wärmekapazität ist. Die Hypothese hat sich also als eine für das Untersuchen des Wärmeaustausches in Gasströmen sehr günstige erwiesen.

Durch den Vergleich von (1) und (4) findet man

$$\mu=\rho l^2\frac{\partial u}{\partial r}. \tag{5}$$

Ferner wird nach Vollziehung der Berechnungen festgestellt

$$\lambda=kc_p\rho r_0 v_*\left(1-\frac{r}{r_0}\right). \tag{6}$$

Nach Einsetzung der Werte λ und u in die bekannte Gleichung der Wärmeübergabe wird

$$\frac{\partial}{\partial r}\left[\frac{r}{r_0}\left(1-\frac{r}{r_0}\right)\frac{\partial T}{\partial r}\right]=\frac{1}{kR_*}\left[R_m+\frac{R_*}{k}\ln\left(1-\frac{r}{r_0}\right)\right]\left(\frac{r}{r_0}\right)\frac{\partial T}{\partial z} \tag{7}$$

worin $R_m=\frac{u_m r_0}{\nu}$, $R_*=\frac{v_* r_0}{\nu}$ — die Parameter von Reynolds, bezogen auf die maximale bzw. dynamische Geschwindigkeit, und ν — die kinematische Zähigkeit sind. Es seien folgende Grenzbedingungen gestellt: $T|_{r=r_0}=0$ und $\left.\frac{\partial T}{\partial r}\right|_{r=0}=0$. Nach Einsetzung von $T=e^{-\omega z}f(x)$ verwandelt sich die Gleichung (7) in:

$$\frac{a}{dx}\left[x(1-x)\frac{df}{dx}\right]+\omega^*\left[R_m+\frac{R_*}{k}\ln(1-x)\right]xf=0. \tag{8}$$

Hier sind $x=\frac{r}{r_0}$ und $\omega^*=\frac{\omega r_0}{kR_*}$ — ein unbestimmter Parameter; dementsprechend werden die Grenzbedingungen $f(1)=0$ und $f'(0)=0$ hervortreten. Eine angenäherte Lösung der Gleichung (8) kann nach dem Verfahren von Ritz erhalten werden, indem man die Aufgabe auf ein Variationsproblem zurückführt:

$$\lim_{\varepsilon\to 0}\int_0^{1-\varepsilon}\left\{x(1-x)\left(\frac{df}{dx}\right)^2-\omega^*\left[R_m+\frac{R_*}{k}\ln(1-x)\right]xf^2(x)\right\}dx=\min. \tag{9}$$

Es sei gesetzt

$$f(x)=a_1(1-x^2)+a_2(1-x^2)^2+\ldots \tag{10}$$

Begnügt man sich mit den zwei ersten Annäherungen von (10), so findet man den Wert der ersten charakteristischen Zahl, nämlich

$$\omega_0=\frac{1.2k^2R_*}{kR_m-0.7R_*}\frac{1}{d}.$$

Ausführliche Berechnungen zeigen, dass wenn man sich mit einer einzigen Annäherung begnügt, so unterscheidet sich der Wert von ω_0 wenig vom denjenigen, der sich bei zwei Annäherungen ermitteln lässt, und folglich ändert sich der Wert ω_0 im geringen Masse, wenn eine genügend grosse Anzahl von Annäherungen berücksichtigt wird. Die bekannte Beziehung zwischen der Maximal- und Mittelgeschwindigkeit gestattet es eine Abhängigkeit zwischen den Parametern von Reynolds in Gestalt von $R_m = Re + 3.75R_*$ festzustellen, was die Möglichkeit gibt in den Ausdruck für ω_0 den auf die Mittelgeschwindigkeit bezogenen Parameter $Re = \frac{\bar{u}r_0}{\nu}$ einzuführen. Nach der von uns abgeleiteten Formel (4) $N_\infty = \frac{Pe \cdot d}{4} \omega_0$ erhalten wir den Grenzwert für Nusselt's Parameter in der Gestalt

$$N_\infty = \frac{0.12\, Pe R_*}{2Re + 4R_*}. \qquad (11)$$

Unter Benutzung der bekannten Beziehung $\tau_0 = \frac{\xi \rho \bar{u}^2}{8}$, worin ξ die Widerstandszahl ist, kann man die Formel (11) in folgender Weise darstellen:

$$N_\infty = 0.12\, Pe \cdot \eta. \qquad (12)$$

Hier sind $\eta = \frac{\sqrt{\xi}}{2\sqrt{8} + 4\sqrt{\xi}}$ und $\sqrt{\xi} = \frac{R_*}{\sqrt{8}\, Re}$.

Die elementare Wärmeaustauschestheorie ([4],[5]) führt zu einer etwas anderen Abhängigkeit zwischen der Nusseltschen Kennzahl und der Widerstandszahl, und zwar

$$Nu = 0.125\, Pe \cdot \xi. \qquad (13)$$

Dieselbe Beziehung wiedergibt im Allgemeinen auch den Grenzwert für Nusselt's Parameter, der aus der Lösung von Latzko ([2]) erhalten wird, in der nur, dem Gesetze von Blasius gemäss, $\xi = \frac{0.316}{Re^{1/4}}$ einzusetzen ist.

Zum Vergleich der Werte ξ und η führen wir eine Tabelle an. Hier sind die Berechnungen in dem Bereich der Reynolds'schen Zahlen über 10^5 nach der Formel von Nikuradze ([1]) durchgeführt: $\xi = 0.0032 + \frac{0.221}{Re^{0.237}}$

Bereich des Gesetzes von Blasius			Bereich des Gesetzes von Nikuradze		
Re	ξ	η	Re	ξ	η
10^4	0.03160	0.02792	10^5	0.01763	0.02146
			$5 \cdot 10^5$	0.01206	0.01801
$4 \cdot 10^4$	0.02235	0.02390	$10 \cdot 10^5$	0.01162	0.01784
			$15 \cdot 10^5$	0.01079	0.01711
$9 \cdot 10^4$	0.01824	0.02185	$20 \cdot 10^5$	0.01046	0.01686

Aus der angeführten Tabelle ist leicht zu ersehen, dass im Wirkungsbereich des Gesetzes von Blasius die Formel (12) gut mit der Formel (13) übereinstimmt. Für grosse Reynolds'sche Zahlen erhält man mit diesen Formeln merklich divergierende Resultate, wobei nach der Formel (12) ein weniger ausgesprochener Abfall des Wärmeaustausches zu beobachten ist.

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1940. Volume XXIX, № 4

PHYSIQUE APPLIQUÉE

SUR LE CALCUL DE LA PUISSANCE DE L'ÉMISSION ÉLECTROMAGNÉTIQUE

Par A. G. ARENBERG

(Présenté par V. F. Mitkevitch, de l'Académie, le 15.VI.1940)

1. Dans les communiqués précédents *, consacrés à la détermination de la puissance de l'émission électromagnétique d'un oscillateur élémentaire, le vecteur de Poynting ($\overline{S}$) était considéré comme somme de deux vecteurs. Le premier ($\overline{S}_v$) comprenait la composante du champ électrique ($\overline{E}_v$) ne dépendant que du potentiel vectoriel $\overline{A}$, tandis que le second ($\overline{S}_s$) — la composante du champ électrique ($\overline{E}_s$), n'étant fonction que du potentiel scalaire φ.

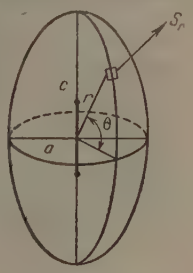

Fig. 1.

On déterminait la puissance émise en intégrant les flux de vecteurs composants du vecteur de Poynting suivant les différentes surfaces enveloppant l'oscillateur. Maintenant nous allons généraliser les résultats obtenus et nous ferons voir que la même chose, comme cas particulier, peut être calculée en intégrant le flux du vecteur $\overline{S}$ suivant la surface d'un ellipsoïde de révolution comprenant l'oscillateur.

2. En effet, dans ce cas (fig. 1) on peut obtenir pour les composantes normales du vecteur Poynting les expressions suivantes:

$$\left.\begin{aligned} S_{vn} &= \frac{\omega M_0^2 \cos\theta}{4\pi r^2 \sqrt{1+m^4 \operatorname{tg}^2\theta}} \left[\frac{k^3}{2} + \frac{k^3}{2}\cos 2(\omega t - kr) + \frac{k^2}{2r}\sin 2(\omega t - kr)\right] \\ S_{sn} &= \frac{\omega M_0^2 \cos\theta}{4\pi r^2 \sqrt{1+m^4 \operatorname{tg}^2\theta}} \left\{[\cos^2\theta + m^2 \sin\theta] \cdot \left[\frac{k^2}{2r}\sin 2(\omega t - kr) - \right.\right. \\ &\quad - \frac{k}{r^2}\cos 2(\omega t - kr) - \frac{1}{2r^3}\sin 2(\omega t - kr)\left.\right] + (m^2 - 1)\sin^2\theta \cdot \\ &\quad \cdot \left[\frac{k^3}{2} + \frac{k^3}{2}\cos 2(\omega t - kr) - \frac{2k}{r^2}\cos 2(\omega t - kr) - \right. \\ &\quad \left.\left. - \frac{1}{2}\left(\frac{2}{r^3} - \frac{3k^2}{r}\right)\sin 2(\omega t - kr)\right]\right\} \end{aligned}\right\} \quad (1)$$

où $m = \frac{a}{c}$ est le rapport des axes de l'ellipsoïde.

* Voir C. R. Acad. Sci. URSS, XXIII, No. 4 et XXIV, No. 9 (1939). La notation a été gardée la même.

En définissant les flux des composantes $\overline{S}$ au moyen de la surface de l'ellipsoïde, nous obtenons pour les valeurs moyennes de la puissance émise, relativement à chacune des composantes, les expressions:

$$\left.\begin{aligned} P_{\Sigma v} &= \frac{1}{T}\int_0^T dt \oint_Q S_{vn}\, dQ = \frac{\omega k^3 M_0^2}{4}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^3\theta}{\cos^2\theta + m^2\sin^2\theta}\, d\theta \\ P_{\Sigma s} &= \frac{1}{T}\int_0^T dt \oint_Q S_{sn}\, dQ = \frac{\omega k^3 M_0^2}{4}(m^2-1)\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^3\theta\cdot\sin^2\theta}{\cos^2\theta + m^2\sin^2\theta}\, d\theta. \end{aligned}\right\} \quad (2)$$

Il est facile de voir que la puissance totale émise, définie par la somme de ces expressions, est égale pour m quelconque:

$$P_{\Sigma} = \frac{\omega k^2 M_0^2}{3} \quad (3)$$

ce qui correspond à l'expression classique, trouvée par Hertz (1).

On peut, par conséquent, présenter les expressions (2) sous la forme:

$$P_{\Sigma v} = \frac{\omega k^3 M_0^2}{3} N \quad \text{et} \quad P_{\Sigma s} = \frac{\omega k^3 M_0^2}{3}(1-N)$$

où

$$N = \left\{\begin{array}{ll} \frac{3}{2}\left[\frac{1}{1-m^2} + \frac{m}{(m-1)^{3/2}}\operatorname{arctg}\sqrt{m^2-1}\right] & \text{pour } m > 1 \\ 1 & \text{pour } m = 1 \\ \frac{3}{4}\left[\frac{2}{1-m^2} + \frac{m^2}{(m^2-1)\sqrt{1-m^2}}\ln\frac{1+\sqrt{1-m^2}}{1-\sqrt{1-m^2}}\right] & \text{pour } m < 1. \end{array}\right\} \quad (4)$$

On voit sur la fig. 2 les résultats des calculs effectués d'après les formules (4), donnant N en fonction de m.

Il est évident que si l'on substitue les valeurs de N pour $m = 0$; 1 et ∞ dans les expressions de $P_{\Sigma v}$ et $P_{\Sigma s}$, on arrivera aux valeurs obtenues antérieurement en intégrant les flux des composantes du vecteur $\overline{S}$ suivant un cylindre, une sphère et deux plans parallèles infinis.

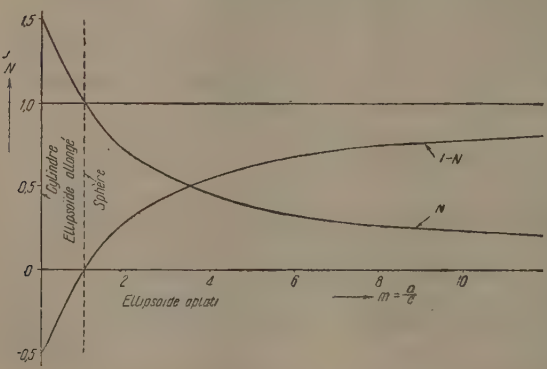

Fig. 2.

3. Les résultats obtenus montrent que les rôles relatifs des potentiels $\overline{A}$ et φ dans le calcul de la puissance varient suivant la forme de surface d'intégration. Cela peut s'expliquer par les raisons suivantes:

a) Le front de l'onde représente, à une distance suffisante de l'oscillateur, une surface sphérique s'élargissant à une vitesse égale dans toutes les directions radiales.

b) La composante invariable du flux d'énergie, allant dans toutes les directions radiales, n'est conditionnée que par la composante du

champ $\overline{E}_v$, car une autre composante du champ $\overline{E}_s$ ne produit dans ces directions que les flux d'énergie pulsants.

c) Dans les directions tangentielles les potentiels $\overrightarrow{A}$ et φ déterminent les flux d'énergie (pulsant dans les plans de méridiens), dont les composantes invariables s'annullent réciproquement.

d) En intégrant par rapport au temps les valeurs moyennes des flux $\overline{S}_v$ et $\overline{S}_s$ suivant une sphère (la surface d'intégration coïncide avec celle des phases égales), on trouve que les flux tangentiels d'énergie sont nuls. En intégrant les valeurs moyennes des flux $\overline{S}_v$ et $\overline{S}_s$ suivant des surfaces fermées quelconques, distinctes de la sphère (dans notre cas un ellipsoïde), les composantes tangentielles de $\overline{S}_v$ et de $\overline{S}_s$, en se projettant sur la normale à cette surface, déterminent certains flux d'énergie, dont le signe dépend de la forme de la surface d'intégration. La somme de tous les flux $\overline{S}_v$ et $\overline{S}_s$, comme cela se comprend, est égale à P_Σ, indépendamment de la forme de surface d'intégration.

e) Lorsque l'on intègre les flux $\overline{S}_v$ et $\overline{S}_s$ suivant une surface cylindrique (ellipsoïde allongé pour $m \to 0$), les projections de ces vecteurs sur la normale sont telles que les énergies, dues aux potentiels $\overrightarrow{A}$ et φ et déterminées au moyen des intégrateurs suivant cette surface, se rapportent comme 3 : 1 (Fig. 2). On trouve le même rapport en calculant $P_{\Sigma v}$ et $P_{\Sigma s}$ d'un conducteur cylindrique par la méthode des forces électromotrices induites *. Cette coïncidence s'explique par le fait que la surface cylindrique d'intégration tend, lorsque son rayon diminue indéfiniment, vers la surface cylindrique du conducteur.

f) En étendant ces résultats au cas d'un oscillateur sphérique **, on peut supposer, qu'on pourra, en calculant P_Σ d'un tel oscillateur par la méthode des f. é. m. induites, ne tenir compte que du potentiel vectoriel $\overrightarrow{A}$, et dans le cas d'un émetteur en forme de disque (un ellipsoïde aplati avec $m \to \infty$) que du potentiel scalaire φ.

L'auteur tient à exprimer sa reconnaissance à N. F. Mitkevitch, de l'Académie, et à B. A. Wwedensky, membre correspondant de l'Académie, pour les conseils et observations reçus au cours de l'exécution de cet ouvrage.

Section des Sciences Techniques.
Commission des Communications Électriques.
Académie des Sciences de l'URSS.

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CHEMISTRY

VIOLATION OF THE LAW OF SIMILARITY WITH PLASTIC DEFORMATION OF MAGNESIUM ALLOYS

By S. I. GUBKIN

(Communicated by N. S. Kurnakov, Member of the Academy; 25.VI.1940)

In the theory of plastic deformation, great significance belongs to the law of similarity established by Fr. Kick (Das Gesetz der proportionalen Widerstände, Leipzig, 1885). According to this law, bodies are held similar if with the ratio of their linear dimensions $a\left(\frac{H_1}{H_2}=a\right)$ their areas are related as $a^2\left(\frac{F_1}{F_2}=a^2\right)$ and their volumes as $a^3\left(\frac{V_1}{V_2}=a^3\right)$. The essence of this law consists in the fact that if the ratio of linear deformations $\left(\frac{\Delta H_1}{\Delta H_2}\right)$ of similar bodies is equal to a, then the ratio of stresses necessary for producing these deformations and the ratio of strain energies will be equal to $a^2\left(\frac{P_1}{P_2}=a^2\right)$ and $a^3\left(\frac{Ag_1}{Ag_2}=a^3\right)$, respectively. There is no difficulty in proving this law on the basis of present-day conceptions on plastic deformation. In fact, with linear extension or compression, strain energy Ag can be represented by the expression:

$$Ag = KV \ln \frac{H}{h}.$$

In considering compression: H is height of body before compression; h — height of body after compression; V — volume of the body; K — mean resistance to deformation in the process of compression; $\Delta H = H - h$ — linear deformation of compression.

Let us assume that we have two similar bodies. The expression of strain energy for the first body can be written as

$$Ag_1 = K_1 V_1 \ln \frac{H_1}{h_1}$$

and for the second

$$Ag_2 = K_2 V_2 \ln \frac{H_2}{h_2}.$$

We have assumed these bodies to be similar; let the ratio of their linear dimensions be $\frac{H_1}{H_2}=a$ and the ratio of their volumes $\frac{V_1}{V_2}=a^3$.

Let us prove now that the energies expended on obtaining linear

deformations which are related to one another as $a\left(\frac{\Delta H_1}{\Delta H_2}=a\right)$ should be related as $a^3\left(\frac{Ag_1}{Ag_2}=a^3\right)$. Indeed

$$\ln\frac{H_1}{h_1}=\ln\frac{H_1}{H_1-\Delta H_1};\qquad \ln\frac{H_2}{h_2}=\ln\frac{H_2}{H_2-\Delta H_2};$$

since $\frac{H_1}{H_2}=a$ and $\frac{\Delta H_1}{\Delta H_2}=a$, so

$$\ln\frac{H}{h_1-\Delta H_1}=\ln\frac{H_2 a}{h_2 a-\Delta H_2 a}=\ln\frac{H_2}{h_2}=\ln\frac{H_1}{h_1}$$

i. e. the relative deformations of these bodies in one of the main directions are equal and consequently should be equal also the mean deformation resistances, i. e. $K_1=K_2$. In this case

$$\frac{Ag_1}{Ag_2}=\frac{V_1}{V_2}=a^3.$$

When there is no similarity of bodies, the proportionality law will be retained, according to which for equal relative deformations $\ln\frac{H_1}{h_1}=\ln\frac{H_2}{h_2}$ (consequently $K_1=K_2$) strain energies are related in the same way as their volumes, i. e.

$$\frac{Ag_1}{Ag_2}=\frac{V_1}{V_2}.$$

Extensive work on the plastic properties of magnesium alloys is being conducted at present at the Laboratory for Mechanical Tests, of the Institute for General and Inorganic Chemistry of the Academy of Sciences of the USSR. In the process of this work there arose doubts whether the proportionality law holds good in the case of magnesium deformation.

The matter was put to the test in special experiments in which cylinders of magnesium were upset with a ram. The cylinders were of two different dimensions: $H_1=20$mm, diameter $D_1=15$ mm, volume $V_1=3530$ mm³; and $H_2=10$ mm, diameter $D_2=15$ mm, volume $V_2=1765$ mm³.

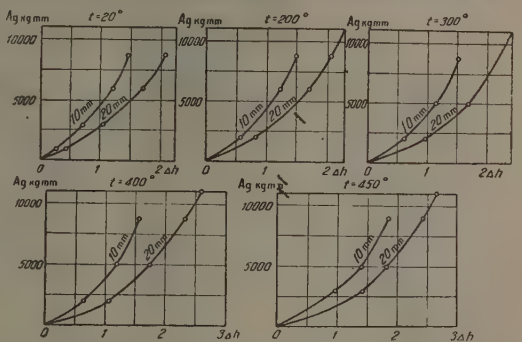

Fig. 1. Dependence between strain energy and upsetting of a test-piece of magnesium.

The blow was struck from a height of 1 m by rams of varying weight. The energy lost in the impact was duly allowed for. Temperatures of 20, 200, 300, 400 and 450° were used in the tests. In the case of more than 20° the cylinders were from the very outset placed in a furnace mounted on the anvil.

Here they were heated to the requisite temperature. Then the furnace was drawn aside, and the ram struck the test-piece. The upsetting effect was measured, and as a result of these measurements curves were plotted for different strain energies (Fig. 1).

In conformity with the above-stated, relative deformations may be equal in the case when the ratio of linear deformations is equal to the ratio of linear dimensions, i. e. if

$$\frac{\Delta H_1}{\Delta H_2}=\frac{H_1}{H_2}=a,\ \text{then}\ \ln\frac{H_1}{h_1}=\ln\frac{H_2}{h_2}.$$

In our case $\frac{H_1}{H_2} = \frac{20}{10} = 2$. The relative deformations will be equal when $\frac{\Delta H_1}{\Delta H_2} = 2$. In this case $K_1 = K_2$ and, according to the proportionality law,

$$\frac{Ag_1}{Ag_2} = \frac{V_1}{V_2} = 2.$$

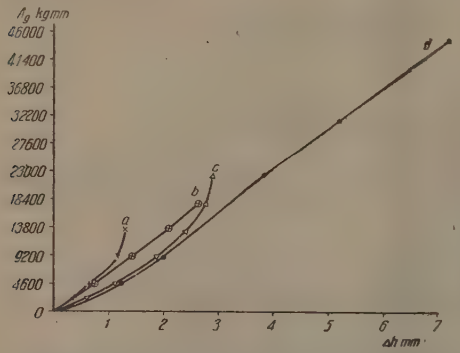

Fig. 2. Dependence between strain energy and upsetting of a test-piece of the alloy MA-3. Curve *a* is plotted for a sample 10 mm in height at a deformation temperature of 20°. Curve *b* is plotted for a sample 20 mm in height at a deformation temperature of 20°. Curve *c* is plotted for a sample 10 mm in height at a deformation temperature of 400°. Curve *d* is plotted for a sample 20 mm in height at a deformation temperature of 400°.

Taking our starting point in the magnitude of deformations for which curves are represented in Fig. 2, we deemed it expedient to perform calculations for a relative deformation of 10%. By the way, this deformation is very frequently available under industrial conditions in the forging of alloys.

Assuming $\Delta H_1 = 2$ mm, we get $\Delta H_2 = 1$ mm. Now here is a table in which with 10% relative deformation the relation of energies is given for every temperature as obtained on the basis of experimental data illustrated by curves in Fig. 1 and from calculations according to the proportionality law.

Table 1

t°C	$\frac{Ag_1}{Ag_2}$ according to experim. data	$\frac{Ag_1}{Ag_2}$ according to proportion. law
20	$\frac{8400}{4700} = 1.79$	2
200	$\frac{8400}{4600} = 1.825$	2
300	$\frac{7400}{4300} = 1.72$	2
400	$\frac{6700}{3800} = 1.76$	2
450	$\frac{5950}{2950} 2$	2

Table 2

t°C	Relative deformation	$\frac{Ag_1}{Ag_2}$ according to experim. data	$\frac{Ag_1}{Ag_2}$ according to proportion. law
20	10%	$\frac{14000}{9000} = 1.55$	2
400	10%	$\frac{9000}{4700} = 1.91$	2
400	30%	$\frac{37000}{24000} = 1.54$	2

On reference to Table 1 it will be seen that the proportionality law is broken for all temperatures but one: 450° C.

A similar departure from the law was also noticed for the magnesium alloy MA-3 (Al—5.6%; Zn—0.99%; Mn—0.33%). The test-pieces were taken of the same dimensions as in the case of magnesium and they were subjected to upsetting at different temperatures in a similar way. In Fig. 2 are plotted the curves for temperatures of 20 and 400° C.

On the ground of experimental evidence, as illustrated by the curve of Fig. 2, Table 2 has been drawn up for relative deformations of 10 and 30%.

In this way a divergence from the proportionality law could be stated for the MA-3 alloy as well.

The established departures from the proportionality law called for verifying the similarity law at room temperature for the alloy MA-3, which

is one of the most widespread magnesium alloys. For this purpose similar test-pieces were prepared. These were of the following two types: $H_1 = 32$ mm, $H_2 = 20$ mm, $D_1 = 24$ mm and $D_2 = 15$ mm.

In this case $\frac{H_1}{H_2} = \frac{32}{20} = 1.6$; $\frac{V_1}{V_2} = 4.095 = (1.6^3) = a^3$.

The samples underwent deformation by the impact of rams of various weights falling from a height of 0.5 m (mean deformation velocity 1.6 m/sec.) and 1 m (mean deformation velocity 2.7 m/sec.).

The results of deformations caused by blows from 0.5 m and from 1 m are represented in Fig. 3 and Fig. 4, respectively. In each of these figures three curves, *a*, *b* and *c*, are plotted. Curves *c* were obtained on the ground of experimental data for test-piece 20 mm high, and curves *b*—also experi-

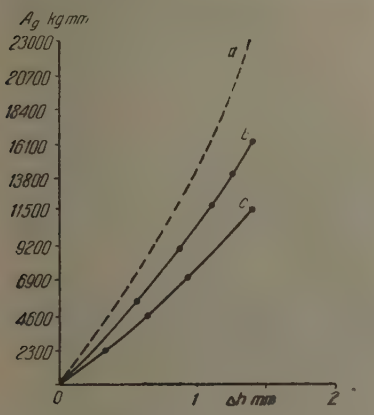

Fig. 3. Dependence between strain energy and upsetting for a test-piece of the alloy MA-3 in the case of the blows delivered from a height of 0.5 m. Curve *c* is obtained experimentally for a test-piece 20 mm high. Curve *b* is obtained experimentally for a test-piece 32 mm high. Curve *a* is obtained by means of calculation for a sample 32 mm high.

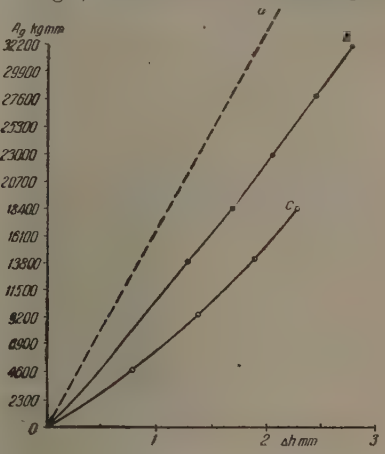

Fig. 4. Dependence between strain energy and upsetting for a test-piece of the alloy MA-3 for a height of fall of ram of 1 m. Curve *c* is obtained experimentally for a test-piece 20 mm high. Curve *b* is obtained experimentally for a test-piece 32 mm high. Curve *a* is obtained by means of calculation for a test-piece 32 mm high.

mentally for test-pieces 32 mm high. Curves *a* were constructed for test-pieces of 32 mm by way of calculations based on the similarity law. If the upsetting ratio between the big and small test-piece be $a = 1.6$, then the ratio of the energies expended on obtaining these upsettings should be 4.095 (a^3). In this way, it is possible, after the experimental curve *c* for the small test-piece to plot curve *a* for the big test-piece.

The given curves exhibit most conclusively the violation of the similarity law in the case of linear upsetting at room temperature of the alloy MA-3. It may be remarked that the departure from the similarity law will be more pronounced for a higher mean velocity of deformation (cf. the curves in Figs. 3 and 4).

It is also significant that the mentioned departure from the similarity and proportionality laws is observable only with comparatively high deformation rates, while at small deformation rates the digression fails to be observable. The reason for this breaking of the similarity law is to be sought in a rise in temperature of the test-piece during the deformation process. The faster runs the deformation process, the higher the temperature of the

specimen will rise and consequently the less becomes the resistance to deformation and the energy outlay. Rise in temperature should depend on rate of deformation, properties of investigated material and correlation of dimensions of test-piece, i. e. on ratio of surface to volume. The greater this relation, the less is the effect of temperature rise, since there are more possibilities for the evolved heat to dissipate into the surrounding medium. The smaller the temperature effect, the less is the violation of the similarity law. This statement must evidently hold true for all metals and alloys, wherefore a check of the obtained conclusions is at present under way at the Laboratory of Mechanical Tests of the Institute of General and Inorganic Chemistry of the Academy, copper, aluminium and Armko-iron being used as test-materials.

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CHEMISTRY

CONCERNING THE STRUCTURE OF PRODUCTS OF ADDITION OF METAL HALIDES TO UNSATURATED COMPOUNDS

By S. L. VARSHAVSKY

(Communicated by A. N. Frumkin, Member of the Academy, 31.VII.1940)

In an article by R. H. Freidlina and A. N. Nesmeyanov (published in this journal, vol. XXVI, No. 1) «On the Structure of the Products of Addition of Metal Halides to Unsaturated Compounds», the authors deal particularly with the problem of the structure of the compounds

$$C_2H_2 \cdot HgCl_2 \text{ (I)} \quad \text{and} \quad (C_2H_2)_2 \cdot HgCl_2 \text{ (II)}$$

As proof of the «main-valency» structure of these compounds they give only the reaction between these compounds and a solution of iodine in ether, which produces chloriodoethylene according to the equation:

$$ClCH = CH \cdot HgCl + I_2 \rightarrow ClCH = CHI + HgClI$$
$$(ClCH = CH)_2 Hg + 2I_2 \rightarrow 2ClCH = CHI + HgClI$$

We can set forth some of our observations which offer supplementary proof of the existence of «main-valency» structure in these compounds. If compound (I) is acted upon with conc. hydrochloric acid in the cold, or with dilute hydrochloric acid heated, it reacts quantitatively according to the equation:

$$ClCH = CH \cdot HgCl + HCl \rightarrow ClCH = CH_2 + HgCl_2$$

However, the formation of vinyl chloride might be ascribed to the fact that the acetylene, generated under the action of hydrochloric acid, immediately reacts with hydrogen chloride in the presence of mercuric bichloride as catalyst, forming vinyl chloride.

But the next experiment shows that the vinyl chloride grouping preexists in the mercuric compound under study. When (I) was treated with 40% hydrobromic acid in the cold, vinyl chloride was generated and was identified as $CHClBr \cdot CH_2Br$, b. p. 160—165°.

In our work we prepared compounds (I) particularly in the following manner: mercuric bichloride was dissolved to saturation in a saturated aqueous solution of NaCl, and acetylene was passed through the reaction mixture thus obtained. The rest of the work was conducted as described in Freidlina and Nesmeyanov's article.

These observations are a part of our work devoted to studying the interaction between acetylene and mercury salts, which was performed with the collaboration of I. H. Shenfinkel and G. G. Ivanov, and which we intend to publish in the near future.

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BIOCHEMISTRY

STRUCTURAL RELATIONSHIP OF SOME CARCINOGENIC AND POLYPLOIDOGENIC COMPOUNDS

By A. SHMUCK and A. GUSSEVA

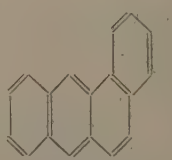
(Communicated by A. A. Richter, Member of the Academy, 9.IX.1940)

The effect of carcinogenic compounds on animal tissues is known to have its most distinctive feature in the disturbance of cell division processes, as a result of which malignant tumors arise. A parallel case is the action of polyploidogenic compounds on plant tissues. These substances, discovered of late ([1–6]), have also been found to disturb cell division and induce tumor-like swelling, out of which chimeric parts containing polyploid sets of chromosomes eventually appear in plants.

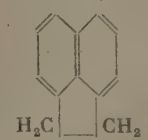
In a certain degree the analogy between these phenomena in plants and animals extends to histological and cytological modifications ([7]), and the deviation of biochemical processes from their normal course in the affected tissues bears considerable resemblance, too ([8]). But the most striking analogy is observed when the biological activity of carcinogenic and polyploidogenic compounds is compared with reference to the particulars of their chemical structure.

Among the polyploidogenic substances discovered by us ([3]) acenaphthene shows the highest biological activity, while naphthalene is the least active. When the latter is made to act on the plants, the formation of two-nuclear cells is often observed, but polyploid sets of chromosomes appear only occasionally. According to P. Gavaudan ([9]), naphthalene, though endowed with unmistakable polyploidogenic activity, is far less active than acenaphthene. To find a similar case among the carcinogenic compounds, we can compare it with 1. 2-benzanthracene, while the strong action of acenaphthene parallels that of cholanthrene.

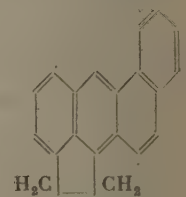
Naphthalene
Weak polyploidogenic activity



1. 2-benzanthracene
Weak carcinogenic activity



Acenaphthene
Strong polyploidogenic activity



Cholanthrene
Strong carcinogenic activity

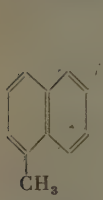
Structural correspondence between these compounds is obvious: naphthalene and acenaphthene form the left part of the molecule of benzanthracene and cholanthrene, respectively.

It is known that the weak carcinogenic activity of the 1.2-benzanthracene is markedly affected by introduction of methylic groups into the mole-

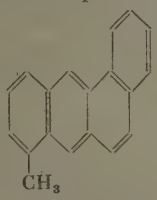
cule of this compound. In which direction it changes, depends upon the place of entry of the methylic group. If it comes to replace one hydrogen atom in the 5th position, the result will be a very active substance, but if hydrogen is replaced in the 7th position, all activity is lost.

Quite analogous instance is furnished by the introduction of the methylic group into the naphthalene molecule: α-methyl naphthalene is very active, while β-methyl naphthalene is altogether inactive.

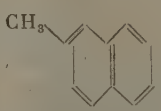
Structural comparison of substances arising from such substitution will reveal an unmistakable parallelism.



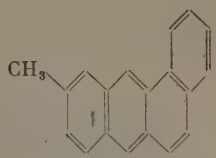
α-methyl naphthalene
Very active



5-methyl-1.2-benz-anthracene
Very active



β-methyl naphthalene
Inactive



7-methyl-1.2-benzanthracene
Inactive

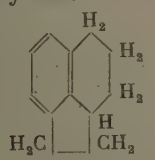
Typical carcinogenic substances, as benzpyrene and methylcholanthrene, lose their activity in response to slightest alterations of their molecular structure ([10]). It will suffice to introduce one hydroxyl group in order to deprive these derivatives of their polyploidogenic activity. The latter disappears also from carcinogenic hydrocarbons when sulpho, amino, nitro or carboxyl groups are introduced.

Among the respective derivatives of our polyploidogenic compounds (acenaphthene and naphthalene) there is again a complete similarity of behaviour.

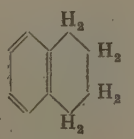
Groups introduced	Carcinogenic compounds (benzanthracene and cholanthrene)	Polyploidogenic compounds (naphthalene and acenaphthene)
Sulpho	inactive ([11], [12])	inactive*
Nitro	» »	»
Amino	» »	»
Oxy	» ([13])	»
Carboxyl	» ([14])	»
Haloid	some are active ([15])	active (only with certain substitutions)
Cyano	» » » »	do

At long last, it is known that in carcinogenic hydrocarbons the hydrogenation of aromatic rings, even if not complete, leads to removal of biological activity. According to Fieser ([15]), neither hexa-, nor tetra-, nor even dihydromethylcholanthrene have any carcinogenic power.

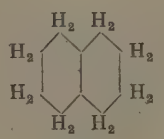
With the hydrogenation of naphthalene and acenaphthene the case is similar. The tetrahydroacenaphthene prepared by us proved to have no polyploidogenic activity whatsoever, and both tetraline and decaline were also completely inactive.



Tetrahydroacenaphthene



Tetraline



Decaline

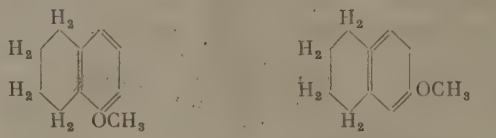
* Information on the polyploidogenic activity of the α-nitronaphthalene is given later.

Consequently, all the investigated methyl derivatives of naphthalene and the functional derivatives of naphthalene and acenaphthene, as well as the hydrogenation products of these compounds, show quite the same behaviour with regard to the polyploidogenic activity as is displayed by the respective derivatives of benzanthracene, cholanthrene and benzpyrene in their carcinogenic activity.

There are some deviations, however, which have not yet been studied at length. These were observed in the biological action and chemical structure of the following compounds. The nitro-derivatives of acenaphthene have no polyploidogenic activity. This is in line with the absence of carcinogenic activity in the nitro-derivatives of cholanthrene and benzpyrene. But on the other hand, according to our previous reference, the α-nitro-naphthalene has proved to be strongly active, unlike its β-isomere.

It will be evident that the polyploidogenic activity of α-nitro-derivative is essentially determined by the position of the nitro-group in the hydrocarbon molecule. It would be of interest to compare the polyploidogenic action of the α-nitro-naphthalene with the carcinogenic action of the 5-nitro-1.2.benzanthracene, but as no data are available on this compound we are not able to draw such comparison.

As has been stated, the hydrogenation of aromatic rings deprives the respective hydrocarbons of their polyploidogenic or carcinogenic activity. It was therefore quite unexpected to find that in the methyl ether of α-naphthol the polyploidogenic activity was not in the least reduced by partial hydrogenation, while the methyl ether of tetrahydro-β-naphthol was not active at all.



Methyl ether of tetrahydro-α-naphthol
Active

Methyl ether of tetrahydro-β-naphthol
Inactive

Among the data available in literature we found the carcinogenic activity reported in certain methyl ethers of oxy-derivatives of carcinogenic hydrocarbons: for instance, in 10-methoxy-1.2-benzanthracene ([15]) and 3-methoxy-1.2-benzanthracene ([16]). All along there were some indications to the absence of such activity in hydroxy and methoxy derivatives, for instance in 3-methoxy-10-methylcholanthrene and 4-methoxy-benzpyrene ([13]).

So the carcinogenic activity seems to depend both on the position of methoxyl groups in the hydrocarbon molecule and on the structure of the nucleus within this molecule. But the polyploidogenic activity of naphthol and naphthoic acid ethers is also determined by the position of the group, and only the α-isomeres of these compounds are active, while the β-isomeres are not ([17]).

It is a matter for regret that in this case, too, we are denied the possibility of comparing the methyl ether of tetrahydro-α-naphthol with the respective derivative of benzanthracene in their biological activity. Perhaps, the total loss of carcinogenic or polyploidogenic activity occurs only with hydrogenated hydrocarbons and not with their functional derivatives.

Malignant tumors are known to be induced even by smallest quantities of carcinogenic compounds, not above thousandths of a milligram. In that particular these compounds are very suggestive of biocatalyzers, such as hormones and vitamins ([10]). No less powerful are the polyploidogenic com-

pounds. The effect of acenaphthene on wheat, for instance, is very strong with some few micrograms of this substance ([18]).

To sum up, whatever particular in the chemical structure and biological action of the naphthalene, acenaphthene and their derivatives was investigated, there was nearly always found complete correspondence with facts known about the carcinogenic substances, the benzanthracene and cholanthrene, in particular.

We do not believe that this might be a purely external likeness. It is more probable to suppose the existence of a general mechanism behind these biological phenomena in plants and animals.

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10. IX. 1940.

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GEOLOGY

DEVONIAN AND CRYSTALLINE ROCKS IN THE WESTERN REGIONS OF THE BASHKIR ASSR

By V. A. BALAEV

(Communicated by A. A. Borissiak, Member of the Academy, 20.VIII.1940).

In June 1940, in the western part of the Bashkir Autonomous Socialist Soviet Republic, at the village Ardatovka, 10 km north-westward from the chief town of the Tuymaza district, when drilling a test-hole in exploration of petroleum, there were discovered at an absolute depth of about 1589 m crystalline rocks, granites, evidently, belonging to the base of the Central Russian platform.

The above-mentioned hole was sunk almost in the roof of a sloping and very extensive geological structure of the brachyanticline type. At the same time it appeared to be situated on the southern slope of a magnetometric maximum of great extent and in the proximity of an isodynamic curve $+400\gamma$.

The crystalline rock disclosed in the terminal stope within the span 1738.2—1739 m has a pinkish colour, consists of large crystals of plagioclase, microcline with micropigmatite structure and vitreous quartz and a small admixture of hornblende. From the arrangement of the dark minerals this rock assumes sometimes a gneissoid aspect. By its composition and structure it is suggestive of syenite-granite.

Immediately above the granite within the span of depths 1735—1738 m lie in ascending order: 1) Grey fine-grained quartz-feldspathic sandstone (occupies nearly half of the interval in the core). 2) Dark greyish argillaceous limestone with rare quartz grains. 3) 0.5 m thick interbed of brown oolitic iron ore. 4) Greenish-grey organogenic detrital limestone with fragments of *Crinoidea*.

The overlying sedimentary rocks to a depth of 1655 m are composed of terrigenous rocks with interbeds of organogenic detrital limestones carrying a fauna characteristic of the Givetian stages of the Middle Devonian. There have been differentiated:

D_2^{2a}—lower beds of alternating layers of grey and greenish-grey clays, shales, grey quartz, sandstones and organogenic detrital limestones containing *Dechenella romanovski* Tschern., *Favosites* sp., *Stromatoporoidea*, whereas the clays seldom carry small scales of fishes. Thickness 30 m.

D_2^{2b}—the upper layers are composed of alternating grey, light-grey quartz sandstones, alveolites and greenish-grey clays and argillites with interbeds of grey, greenish-grey argillaceous limestones. In the limestones occur: *Spirifer subumbonus* Hall., *Productus subaculeatus* Murch., *Schizophoria*

cf. *bistriaia* Tschern., *Atrypa ailinensis* Vern., *Liorhynchus* sp., *Rugosa*. The sandstones show remains of *Psylophyta*. Thickness 53 m.

In the upper parts of these strata prevail fine-grained sandstones and aleurolites. They proved to be markedly oil-stained.

Above rest terrigenous and carbonaceous rocks with fauna characteristic of the Frasnian stage of the Upper Devonian. Among those are to be distinguished:

D_3^{IA}—the horizon under the Domanik beds. 1) Bed-pack of argillaceous rocks, represented by greenish-grey and violet clays and argillites with rare bands of clayey limestones. In these beds *Lingula* sp. has been met. Thickness 15 m. 2) Bed-pack of limestones grey and greenish-grey with interbeds of clayey intercalations and clay bands. Containing: *Spirifer (Cyrtospirifer) disjunctus* Vern., *Productus seriseus* Buch., *Atrypa* cf. *ailinensis* Vern., *Liorhynchus* sp. Thickness 5 m.

D_3^{I}—Domanik beds constituted by dark-grey bituminous shales with interbedded pteropod limestones. Occurring: *Liorhynchus* cf. *mesocostalis* Hall., *Buchiolla retrostriata* Buch., *Giphioceras* sp., *Bactrites* sp., *Tentaculites* sp., *Stiolina* sp. Thickness 30 m.

D_3^{Ic}—the horizon above the Domanik beds represented by grey limestones with interbeds of dolomites, and at the top—brownish-grey, occasionally clayey limestones with small intercalations of somewhat bituminous limy clays and shales. Occurring: *Rhynchonella cuboides* Sow., *Atrypa reticularia* L., *Spirifer pachirinchus* Vern., Sp. *conaideus* Roem., *Lingula* sp., *Manticoceras* sp. and others. Thickness 105 m.

From faunal evidence the overlying Devonian deposits belong to the Famennian stage. In the lower parts they are represented by uniform beds of grey dolomites, in places karstic, at the base sulphatized, with interbeds of anhydrite 2—3 m thick. Recrystallized remains of *Brachiopoda* and *Ostracoda* have been encountered. Thickness 145 m.

Higher up lie beds of grey crystalline granular limestones with karst solution cavities. Occurring: *Spirifer* ex gr. *archiaci* Murch., *Productus lochrimodus* Vern., *Camarotoechia* ex gr. *livonica* Buchn. and others. Thickness 152 m.

The roof of the Upper Devonian is traced by us at a depth of 1203 m (—1053) along the terminal interbeds of limestone abounding in remains of *Camarotoechia* ex gr. *livonica* Buch.

The overlying sedimentary rocks belong to the Carboniferous and Permian. The coal-bearing deposits in this district hardly show any essential distinction from the Tuymaza district adjacent from the south. Both districts have been described by the author on a previous occasion ([5]).

According to literary data ([1,2]) the discovered granites are analogous to the Boguchar granites and evidently belong to pre-Cambrian formations.

By the lithologic character of the rocks and thickness the Devonian deposits drilled in the Tuymaza district have a great deal in common with the sections of the Central Devonian field. In both districts deposits of the Givetian and the lower part of the Frasnian stages show variegated lithologic composition. The dominant rocks in these sections are sandstones, clays, argillites, whereas limestones with marine fauna hold a subordinate position. On the basis of lithological and faunistic similarity one may see among the Devonian of the Tuymaza district analogues of Staro-Oskolsk, Shtchigrov and Semiluki beds of the Central Devonian field. It is noteworthy that this part of the section of the Devonian in the Tuymaza district as compared to the sections of the Central Devonian field is more carbonaceous and contains finer detritus material. The western slope of the Urals shows at that time a prevalence of limestones, whereas clastic rocks are very scantily represented.

Beginning from the middle horizons of the Frasnian stage, the Devonian of the Tuymaza district assumes great likeness with the Devonian of the western slope of the Urals. The Domanik series of the Tuymaza district shows faunal and lithologic affinity with the most typical sections of the Domanik of the western slope of the Southern Urals. Domanik beds are not known in the regions of the Central Devonian field. The Petino beds seem to be their time-equivalents. The overlying deposits of the Tuymaza Devonian bear resemblance to those of the Western Urals, however differ from the latter in greater thickness and sulphatization, though at the same time they have a common faunal character. The Voronezh, Eletzk and Dankovo-Lebedyansk beds of the Central Devonian field are distinguished from their co-eval analogues in the Tuymaza district by smaller thickness and the presence of interbeds of clastic rocks. Evidently, beginning from the Fammenian stage, within the boundaries of the western part of the Central Russian platform was segregated an independent facies, connected with the increased salinity of the sea basin.

The main conclusions may be summarized thus: 1) The crystalline rocks disclosed in the Tuymaza district belong to the pre-Cambrian base of the platform. 2) Marine sedimentary formations in the eastern part of the Central Russian platform, as well as in the regions of the Central Devonian field, date from the second half of the Middle Devonian. 3) In the course of the long continental period the crystalline bed of the platform was disintegrated to a large extent and was the source of detritus accumulation to form the old «unfossiliferous» series in some of the regions of the western slope of Urals. 4) The drift of the detritus in the direction from south-west and west-eastward into the lowland regions adjoining the Ural Mountains. 5) The data obtained may lend some more precision to the paleographic schemes ([3],[4]) of the Devonian known in the literature. 6) The presence of Domanik beds and porous rocks with abundant oil signs in the upper parts of the Givetian stage, disclosed in the Tuymaza district, offer great opportunities for the discovery of new oil-bearing horizons in other regions of the «Second Baku» as well.

In the author's opinion, to such regions are referable in the first place the Samara Bend where oil-bearing deposits are liable to be struck at depths close to or somewhat less than those of the Tuymaza district. However, the nearest to the surface, approximately at a depth of about 1000 m, these horizons may be met in the region of Teplovka in the Saratov province.

There are good prospects for oil beds to be found in the Devonian of the western regions of the Ural lowland as well as on the western slope of the South Ural Mountains, where a number of structures favourable to oil accumulation are known to occur.

Scientific Oil Prospecting Institute.

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GEOLOGY

ON THE AGE OF THE CONTINENTAL DEPOSITS OF THE EASTERN SLOPE OF THE SOUTH URALS

By K. V. NIKIFOROVA

(Communicated by A. N. Zavaritzky, Member of the Academy, 9.VII.1940)

On the eastern slope of the South Urals of rather wide occurrence are continental deposits, arranged in separate patches or belts and often containing gold placers.

In the recently published (1939) geological map of the Urals on a scale 1 : 500000 and in the explanatory note to it the patches or belts of continental gold-bearing deposits are assigned to the Jurassic.

A. L. Yanshin, who is the author of the chapters on the Jurassic and Tertiary continental deposits, in his comments to the map refers to the Jurassic the following areas of development of these deposits. a) Two meridional belts of auriferous sands, covered with thick variegated clays, located north of the Kamyshly-Ayat river near the settlement Novo-Georgievsky. b) The belt of deposits in the upper course of the Suunduk river at the settlement Adrianopolsky, where the series are underlain by a bed of auriferous sand and covered with variegated clays, clayey sands and «beliki» conglomerates. c) A number of gold-bearing placers in the basins of the Suunduk, Tashty-Butak and Gumbeika rivers.

Yanshin has given no consideration at all to the auriferous beds developed in a wide streak in the basin of the Zingeika river north of the village Katzbakhsky and in the basin of the Zhilaya river north-eastward of the village Novinsky; similar deposits on the Sintashta river north and north-eastward of the village Bredy; deposits developed along the left bank of the Kamyshly-Ayat river west of the village Knyazhinsky; patches of similar deposits south of the village Chernigovsky on the Bokhta river.

The work of the geological-geomorphological party of the Uralian expedition of the Institute of Geological Sciences of the Academy of Sciences of the USSR in 1938 and 1939 covered all the above-named regions of distribution of continental auriferous deposits. As a result of this work a composite stratigraphical section of the auriferous deposits could be drawn, its mode of occurrence and origin were made out, and an idea was formed of its age, due to the findings of flora and fauna in the lower parts of the sections.

The continental auriferous deposits are represented by clays and pebblestones of light hues (white, pink, light-grey and yellowish), occurring often on elevations between rivers and composing the basement of the Quaternary terraces and gulches. The described deposits are underlain by limestones of Upper Devonian and Lower Carboniferous age and are confined to sink-holes and solution cavities.

The lower part of the section is composed mainly of areno-pebbly material; the pebblestones interbedded with sands form thick beds—up to 30—50 m. The pebbles up to 15—30 cm in diameter are well rounded and flattened. In the lowermost part, directly upon the limestone «bedrock» lie auriferous sands. The upper part of the section consists of light-grey plastic clays, often with red and ochreous spots, sometimes containing gypsum druses.

In many drill-holes in the lower part of this section (over the auriferous sands or among pebblestones) lenses of light-grey and brown clays with lignite and remains of wood were discovered. The latter, as determined by A. V. Yarmolenko, belong to the species: *Taxodioxylon distichum* Mercl. sp. and *Cupressionoxylon uralense* Jarm encountered in the Neogene deposits of lake Smolin near Chelyabinsk (Sarmatian, Pontian, partly analogous to the second Mediterranean stage), in the Neogene of Abrosimovka on the Tara river, and in a number of other localities of Western Siberia, Kazakhstan, and the western slope of the Urals, where Neogene deposits occur.

According to Yarmolenko, our findings present «a precise analogy to the Smolin horizon of the Neogene». The belt of the auriferous deposits described, developed in the upper course of the Suunduk river at the village Adrianopolsky, was traced by the author without interruption to the settlement Kvarkeno situated on the right bank of this river on a 16 m erosional terrace. The basement of the terrace consists here of limestones of the Upper Carboniferous, and on these, in sink-holes, lie coarse quartz pebblestones interbedded with sands, analogous to those described above. These sands are also auriferous.

The upper horizons of clays are lacking, and the areno-pebblestone series is covered with Quarternary alluvium.

In the lower horizon of pebblestones, at the boundary of gold-bearing sands, a tooth was found, determined by E. J. Belyaeva as that of *Mastodon borsoni*. Higher up, in the second pebblestone horizon, separated from the first by a layer of sand 1 m thick, a tooth of *Elephas meridionalis* was found.

Judging by the fact that the belt of Kvarkeno deposits is a direct continuation of the belt developed in the upper course of the Suunduk, and on the basis of the lithological likeness of the lower part of the section, they may be considered to be of the same age. The flora and fauna described point to Neogene age. It should be also noted that at the southern boundary of the region described on the left bank of the Aidyrli river, among the dark-brown carbonaceous clays, S. N. Naumova has determined spores of coal which she referred also to the Tertiary.

Summing up the aforesaid, it may be stated that all the enumerated patches of continental deposits, shown by Yanshin as «Jurassic» on the geological map of the Urals, published in 1939, should be referred to the Tertiary, namely to the Neogene.

A detailed study of these deposits enabled us to establish that all of them are confined to the old hydrographic network of a meridional direction, i. e. following in the main the trend of strike of the Paleozoic rocks, and not coinciding with the present river network which follows mainly a latitudinal direction.

It is possible that this old Neogene river network inherited the Mesozoic depressions and repeated in the main features the direction of the Mesozoic network. The Mesozoik river deposits were destroyed and washed out, along with the old weathering crust and the sediments of the Cretaceous and Paleogene seas. All these formations furnished material for the deposits of Neogene streams, the preserved patches of which we have just described.

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MINERALOGY

TITANOVESUVIANITE FROM THE PEROVSKITE MINE IN THE CHUVASH MOUNTAINS AT THE SOUTH URALS

By L. L. SHILIN

(Communicated by A. E. Fersman, Member of the Academy, 10.IX.1940)

While inspecting the mines of the Shishim mountains, we also examined the Perovskite mine located 4 km north-east the village Medvedevka in the Chuvash mountains. This mine used to attract the attention of investigators by its beautiful perovskite crystals.

When digging prospecting trenches in the eastern part of the Perovskite mine at the boundary of marmarized limestones with contact rocks, a mine-

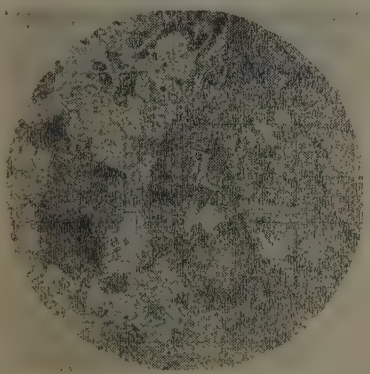

Fig. 1.

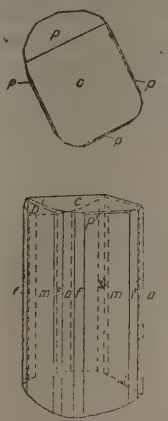

Fig. 2.

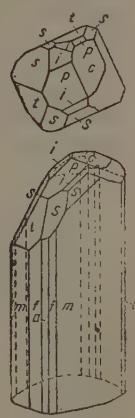

Fig. 3.

ral was discovered of a brownish colour translucent at the edges. Microscopical analysis showed the rock itself to consist mainly of a brownish mineral which proved to be vesuvianite, of diopside, chlorite and a subordinate amount of serpentine. The sequence of segregation of the minerals is as follows: vesuvianite, diopside, chlorite and serpentine (Fig. 1).

The presence in the Perovskite mine of minerals rich in titanium, as perovskite and titaniferous garnet, called our attention to the mineral newly found in this mine. As shown by chemical investigation, it contains up to 4.59% TiO_2 which seems to be rather high.

This mineral can therefore be separated as an independent variety of vesuvianite and named «the titanovesuvianite», along with chromvesuvianite, beryllium-vesuvianite, copper-bearing vesuvianite-cyprine, boron-containing vesuvianite-viluite.

In collecting mineralogical material two types of titanovesuvianite crystals were observed lying in a compact slightly weathered rock, from which they are rather easily extracted. To the first type belong thick prismatic crystals with a well developed pinacoid (001); to the second—thin prismatic crystals of titanovesuvianite with a well developed head with numerous pyramidal faces and a slightly developed pinacoid. The above crystals sharply differ from each other, which made me separate them as different types. Figs. 2 and 3 show crystals of both types.

Out of the material collected the most suitable for measurements proved the crystals shown on the accompanying diagrams, being essentially typical for the habit of the titanovesuvianite from the Perovskite mine. Three of them were measured on Goldschmidt's goniometer.

As a result of the measurements the following 14 forms were established: *c*—(001); *p*—(111); *b*—(221); *t*—(331); *d*—(241); *z*—(121); *s*—(311); *i*—(132); *m*—(110); *a*—(010); φ—(350); ψ—(470); *f*—(120); *h*—(130).

By the number of forms discovered titanovesuvianite is similar to the vesuvianites from the Shishim mountains, Veselkina and Gulenko mines, where S. M. Kurbatov [1] found in hollows of garnet-vesuvianite rocks vesuvianite crystals, among which he distinguished 16 forms: *c*—(001); *p*—(111); *b*—(221); *t*—(331); *i*—(312); *z*—(121); *q*—(833); *s*—(311); θ—(11.33); *y*—(411); *v*—(511); Δ—(15.22); *m*—(110); *a*—(100); *f*—(211) and *h*—(310).

As is seen from the above data, twelve of these forms are common both to the vesuvianite of the Veselkina and Gulenko mines, and to the titanovesuvianite of the Perovskite mine. To these belong the following forms: *c*, *p*, *b*, *t*, *z*, *s*, *i*, *m*, *a*, *f* and *h*. Forms typical of titanovesuvianite from the Perovskite mine alone are *d*, φ and ψ; forms of vesuvianite characterizing the Veselkina and Gulenko mines—*q*, θ, *y*, *v* and Δ.

The table shows the number of faces in each form in per cent for titanovesuvianite.

	c	*p*	*b*	*t*	*i*	*z*	*s*	*d*	*m*	*a*	*f*	*h*	φ	ψ
	001	111	221	331	321	121	311	241	110	010	120	130	350	470
1	1	4	1	3	1	2	2	1	4	3	2	4	1	1
2	1	4	—	—	—	—	—	—	4	4	7	—	—	—
3	1	3	—	3	3	—	4	—	4	4	6	1	—	—
%	100	91.6	8.3	50	16.5	16.3	25	8.3	100	91.7	62.5	33.3	8.3	8.3

Axial ratio $a : c = 1 : 0.5375$. According to Goldschmidt $a : c = 1 : 0.5376$; according to Kurbatov $a : c = 1 : 0.53732$.

The refractive indices of titanovesuvianite measured by immersion method show the following values: $Ne = 1.1733 \pm 0.002$; $No = 1729 \pm 0.0015$; $Ne - No = 0.004$.

Both from its crystallographic forms, refractive indices and axial ratio titanovesuvianite approaches common vesuvianites. The refractive indices for titanovesuvianite are the highest of all known data for vesuvianites, on account of its high TiO_2 content.

As to its genesis, titanovesuvianite may be referred to minerals formed by contact processes. This might possibly have occurred in an environment rich in TiO_2, CaO, Al_2O_3 and Fe_2O_3, along with such titaniferous minerals as perovskite, schörlomite and titanovesuvianite.

Karpinsky Geological Museum.
Academy of Sciences of the USSR.
Moscow.

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GENETICS

POLYPLOID FORMS OF SPRING RYE

By L. P. BRESLAVETZ

(Communicated by A. A. Richter, Member of the Academy, 16.VI.1940)

In the spring of 1939 100 seeds of spring rye were subjected to the action of various colchicine concentrations for different periods of time. The seedlings showed different peculiarities as compared with the control, especially as regards shape and colour of the leaves. 58 seedlings had very wide dark-green leaves which were sometimes very thick. Of this number 24 plants perished at more or less early stages of development, the rest attained maturity and produced seeds. The further development of these seedlings showed a very interesting peculiarity, viz. the second leaves were in the great majority very narrow, sometimes three-four times narrower than the normal, while only the third and sometimes the fourth leaves attained the width of a normal leaf of the given age. Only two seedlings formed an exception, their second leaves being also very wide, however, these seedlings perished very soon.

After the plants began to form ears, the top leaf was cut off and its length and width were measured. A bit of the epidermis was taken from the lower surface, and the stomata were measured. Three plants were chosen according to the size of their stomata. The stomata proved to be double or almost double the size as compared with the control (plants No. 4, 24 and 30). Plant No. 4 proved to be a pure tetraploid, i. e. 28 chromosomes instead of the usual 14 were found in the roots.

As soon as the rye began to flower, the diameter of the pollen grain of the survived plants was measured. Plant No. 4 showed the largest pollen grain; however, in the case of rye it is not always possible to judge of polyploidy by the dimensions of the pollen grain. Among the survived plants chosen at early stages six showed morphological characters of polyploidy: the shape of the first leaves, and the dimensions of the stomata and of the pollen grain (Table 1). One plant proved to be a pure tetraploid. At the beginning of its development it possessed very wide leaves with well expressed venation, the second leaves were narrow, while the length of the last leaf markedly surpassed that of normal leaves; the length and the area of the stomata and the diameter of the pollen grain also was twice that of the control; the root cells had 28 chromosomes. Thus, according to its morphological, anatomical and cytological characters, the plant was a tetraploid.

The first leaves in plant No. 17 were very wide, the stomata were longer than in the norm, the diameter of the pollen grain equaled that of the control. The chromosome number was 21, i. e. the plant was a triploid.

The first and second leaves in plant No. 19 were very narrow. This plant succumbed very soon, though we had time to fix its roots; the counts of the chromosome showed that the plant was an octoploid ($2n = 56$).

Plants No. 24 and 30 had stomata resembling in size those of polyploid plants. But, as the chromosome number was determined in neither of them, these plants cannot be considered as polyploids.

Finally, plant No. 63 had very wide and very thick first leaves and narrow second leaves. The leaves were shorter than those of the controls, while the length of the stomata did not surpass that of the controls. As to the chromosome number, it should be mentioned that 28 chromosomes were found in those few plates which were detected in the roots of this plant. However, owing to the small number of available plates, it was not possible to class this plant with the polyploids; that question cannot be answered, until the chromosome number is determined in the progeny.

Table 1

Morphological Characteristics of the Plants

Action of colchicine	Nos. of plants	Leaves in cm				Stomata*		Diameter of pollen grains*	Chromosome number
		First	Second	Length	Width	Length	Area		
Control, average for 10 plants		normal	normal	21.75	1.13	20.4	205.19	22.85	$2n=14$
0.04%, 48 hours	4	very wide	narrow	25.2	1.0	27.8	426.14	26.02	$2n=28$
			—	17.2	1.2	22.94	—	23.02	$2n=21$
0.02%, 48 »	17	wide							
0.02%, 96 »	19	narrow	narrow	—	—	—	—	—	$2n=56$
0.02%, 96 »	24	wide	very narrow	23.8	0.9	—	353.06	23.90	—
0.04%, 96 »	30	narrow thick,	—	29.5	1.4	25.29	376.84	—	—
0.02%, 5 days	63	wide	narrow	15.2	0.9	19.22	—	22.28	$2n=28$

For the cytological characteristics we give both nuclear metaphase plates and the resting nuclei. When the test-object is well known, the question as to the condition of the diploid or polyploid cells may be practically solved without chromosome counts, since the number and shape of the nucleoli in the resting nucleus are very characteristic. However, the final answer to this question is given by chromosome counts.

The polyploid condition becomes manifest first of all in the size of the cells and nuclei and, as our observations showed, in the shape of the former.

Table 2

Size of Cells and Nuclei Corresponding to the Increase in Chromosome Sets*

Nos. of plants	Condition	Cells			Nuclei		
		Diameters		Ratio longit. to transv.	Diameters		Ratio longit. to transv.
		Longitudinal	Transversal		Longitudinal	Transversal	
2	Diploid	29.9	14.2	2.13	10.7	9.0	1.18
17	Triploid	27.0	17.8	1.52	12.5	10.4	1.19
4	Tetraploid	34.4	17.5	1.96	14.5	11.0	1.32
19	Octoploid	27.5	12.8	2.14	12.5	9.1	1.42

* In divisions of the ocular micrometer.

Besides differences of the cell and nucleus diameters, given the presence of varying chromosome numbers, this table shows the different degree of cell extension as dependent on the polyploid state of the cells. Both diameters of the tetraploid cells are enlarged as compared with the diploid cells. The triploid cells are extended in the transverse direction, while those of the octoploids are extended in the longitudinal direction. Figs. 1, 2, 3 and 4 illustrate these changes.

It is noteworthy, that the tetraploids have the largest cells and nuclei.

The question as to the mechanism of arising of polyploids under the action of colchicine presents no difficulties so far as tetraploids and octoploids are

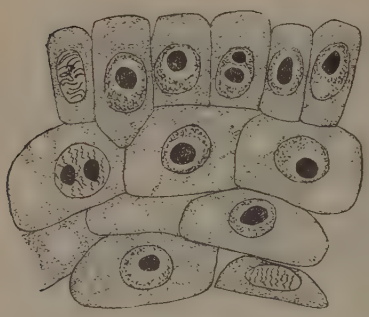

Fig. 1. Part of a cross-section of a diploid rootlet.

Fig. 3. Part of a cross-section of a tetraploid rootlet.

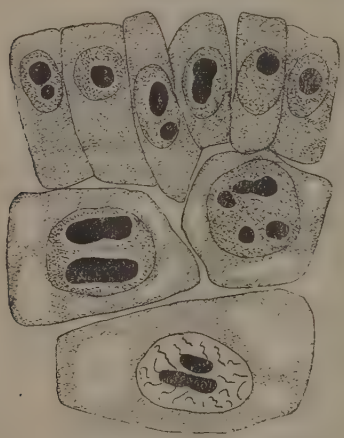

Fig. 2. Part of a cross-section of a triploid rootlet.

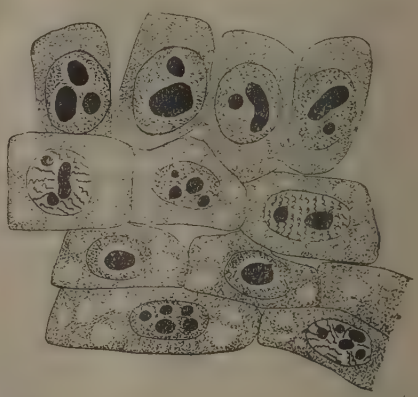

Fig. 4. Part of a cross-section of an octoploid rootlet.

concerned. In the former case it is a single retardation in the division of the protoplast after nuclear division, while in the latter case the stop takes place twice. As regards the triploid plants, the process might possibly have been a more complicate one. The newly arisen tetraploid chromosomes probably happened to be irregularly distributed between the poles in the initial cells, and only those cells were preserved which had received 21 chromosomes each. The possibility of such irregularities has been repeatedly stressed in the literature.

Polyploidy has no effect upon seed formation in spring rye, i. e. the polyploids do not overcome sterility during self-fertilization. The same has been pointed out by us for winter rye.

Table 3 shows that the attempts made to isolate the ears in a true tetraploid resulted in the occurrence of a single seed, while in a triploid plant 5 seeds were produced. Plant No. 63 referred by us to the dubious tetraploids gave 86 seeds after free pollination.

Table 3

Relation Between the Number of Chromosome Sets and Size and Weight of Seeds

Nos. of plants	Condition	Number	Length in mm	Width	Total weight in g.	Weight of one seed
Control	Diploid	100	8.37	2.42	2.60	0.026
4	Tetraploid	1	10.0	2.50	—	0.030
17	Triploid	22+5	10.30	3.01	0.79	0.036
24	Tetraploid ?	10	7.60	2.30	0.25	0.025
30	Tetraploid ?	49	10.10	2.92	2.21	0.045
63	Tetraploid ?	86	9.90	2.90	1.98	0.024

The largest seeds belong to a triploid (Fig. 5). The highest corn weight belongs also to one of the seeds of this triploid. The highest weight was shown by the seeds of plant No. 30 which had very large stomata; its chromosome number is unknown. The size of the seeds approaches that of the polyploids.

The large size of the triploid seeds cannot be explained by the excess of nutrition of a small number of seeds, since the tetraploid seed is somewhat smaller than the triploid one, although the former is unique in the ear.

The increase in size and weight of the seeds gives grounds to assume that the polyploids obtained are of interest as material for selection which we are going to begin next spring.

On the basis of the results obtained the following conclusions may be drawn: 1) The action of definite colchicine solutions may cause the formation of whole polyploid plants in rye, similarly to many other plants. 2) Different degrees of polyploids were obtained, including an octoploid which proved non-viable in our experiment. 3) The occurrence of a triploid chromosome number in the somatic tissue is unusual. 4) The polyploids can be distinguished by their morphological, anatomical and cytological characters prior to maturation. 5) The increase in chromosome number is associated with an increase in cell dimensions both in the root tissues and in the cells of the guard cells of the stomata. 6) The doubling or trebling of the chromosome set is connected with an increase in size and weight of the seeds. 7) Polyploidy does not affect the physiological self-sterility which is inherent in rye.

Fig. 5. Photo of rye corns with different numbers of chromosome sets.

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GENETICS

POLYPLOID RACES IN MILLET (*PANICUM MILIACEUM* L.)

By D. N. ARENKOVA

(Communicated by A. A. Richter, Member of the Academy, 9.VI.1940)

Size of seed in millet is of great economic significance. Hence, it seemed of interest to obtain, by means of autopolyploidy, forms of millet having larger seeds, which, at the same time, would result in increased yield. With this aim in view we adopted, in the spring of 1937, the method of chromosome doubling by colchicine treatment reported by Blakeslee and Avery (1). As material we took two forms of millet, white Chinese 2301 and red Saratov 742, obtained from the Department of Geographical Sowings of the Maikop Experiment Station of the Institute of Plant Industry.

We succeeded in obtaining tetraploid millet both when we treated seeds and when we treated young plants with colchicine. In the first case, dry seeds were soaked for 50—70 hours in β-indol-acetic acid (0.001%) at a temperature of 20—25° C, by the end of which period they germinated. The root-tips were then fastened to slides in a weak sugar solution of 1% agar, while the sprouts, by rotating the slides 180°, were immersed in a 0.1% aqueous solution of colchicine for 24—48 hours. In the second case, plants 5—6 cm in height were treated with a 0.1—0.2% aqueous solution of colchicine, which was applied in drops to the leaf axils by the aid of a syringe. The aim was to apply the colchicine to the buds located in the leaf axils from which the tillers arise. In both cases the growth and development of the treated plants were greatly retarded. At the time of flowering they were only 12—25 cm high, while the control plants were 75 cm high. The inflorescences on the treated plants were very small with only a few spikelets. Very few tillers were produced. On the same plant some panicles had small and others large spikelets. The former gave seeds of normal size for the given variety, the latter—seeds considerably larger than normal.

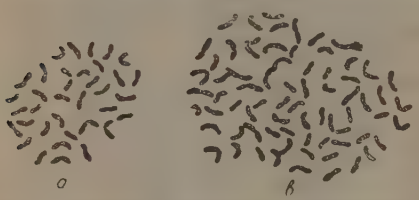

Fig. 1. Nuclear plates of *Panicum miliaceum* (millet): *a*—control, $2n=36$; *b*—experimentally obtained tetraploid, $2n=72$.

In the spring of 1939 a cytological analysis was made of plants grown from the large seeds and from control seeds. According to Avdulov (2), the cells in root-tips of common millet contain 36 chromosomes. In the root-tips of plants grown from large seeds we found, on the other hand, 72 chromosomes (Fig. 1, *a*, *b*). These plants, obtained from large seeds of white Chinese

millet were, therefore, tetraploids. On May 30 we sowed separately in pots in the greenhouse, both the large and the small seeds obtained from the treated plants and also seeds from the control plants. A total of 197 seeds was sown*, they all gave 100% germination. When the plants had attained a height of 4—5 cm, they were set out in the open at a distance of half a meter apart. We made a careful study of 179 of these plants grown in the open.

In Table 1 we give comparative data characterizing diploid and tetraploid millet based on our study of both forms in 1939, the respective figures being means obtained by averaging the measurements of 100 plants of each form (Fig. 2).

Table 1

Character	Diploid millet	Tetraploid millet	Character	Diploid millet	Tetraploid millet
Height of plant . . .	105 cm	115 cm	Length of branches of the first order at base of inflorescence	20 cm	21 cm
Colour of plant . . .	Yellowish-green	Bluish-green	Number of spikelets per panicle	1400	1000
Number of stems per plant	5	4.3	Shape of kernel . . .	Oval, orbicular	Oval, orbicular
Thickness of stem .	Medium	Thick	Colour of kernel . .	Yellowish white	Yellowish-white
Number of internodes	5	5	Absolute weight of 1000 seeds . . .	7.2 g	10.5; 12.5 g
Tillers	Many	Few	Susceptibility to shattering	Medium	Very slight
Pubescence	Medium	Dense	Friability	Friable	Friable
Length of leaf . . .	27 cm	36 cm	Lodging	Highly susceptible to lodging under weight of ripe seeds	Lodging not observed
Width of leaf . . .	2.2 cm	2.4 cm			
Type of panicle and its colour	Drooping, compact, colourless	Drooping, lax colourless			
Length of panicle . .	23 cm	27 cm			
Ratio of number of branches of the first order per unit length of panicle	0.8	0.7	Vegetative period . .	75 days	92 days

The plants obtained from the small seeds produced by the treated plants closely resembled the control plants. Differences in morphological characters between the tetraploid plants and the control plants were noted from the early stages of development. At first the tetraploid plants differed chiefly in their denser pubescence and deeper colour. As the plants developed, the distinguishing characters became more numerous. A comparative analysis of the two types showed that the tetraploid plants constitute a new form differing greatly from the initial form. The panicles of the tetraploid form are very lax, while those of the diploid are compact. This may be explained by the fact that in tetraploid millet there are fewer spikelets per panicle; the panicle is longer, the spikelets are larger, and the branches of the third to fifth orders are less well developed than in the control. The shape of the kernel in both types is usually oval. The absolute weight of 1000 seeds of the tetraploids amounts to 12.5 g, while 1000 seeds of the initial diploid form weigh only 7.2 g (Fig. 3). A few of the tetraploid plants produced seeds slightly smaller

* Including a few seeds sown early in the spring, from which 15 plants were obtained and grown in the greenhouse. Owing to insufficient light, these plants were small and yielded poorly, even those from control seeds.

than the rest and round in shape, weighing 10.5 g per 1000. This fact induced us to make a more careful investigation of the seeds of the initial material. It was found that among the control plants and also in the progeny from the small seeds obtained from the treated plants there were some plants bearing oval and some bearing round seeds. This circumstance, together with the fact that panicles with large seeds were obtained on dozens of treated plants, indicates that the initial material consisted of different morphological and possibly biological types. For this reason the seeds were gathered separately from each tetraploid plant.

Fig. 2. *a*—control; *b*—tetraploid.

As regards the yield and fertility of the tetraploids, we found that they had fewer spikelets per panicle than the diploids. But in diploid millet not all the spikelets give seeds. According to Maisuryan, at the base of the panicle and in its inner part there are found many abortive spikelets and atrophied flowers. We also observed the same phenomena. In the initial form we found from 5 to 7% of abortive spikelets. Failure of seeds to set in normally appearing spikelets occurred in 7 to 9% of the cases. As a result the yield of the control plants was reduced by 15%. From 1400 spikelets, the average number for a control plant, the yield would amount to 1190 seeds, weighing 8.7 g. From 5 to 7% of the spikelets of tetraploid millet are also abortive. If in tetraploid millet the percentage of seeds set corresponded to that in the initial form, then on a panicle of tetraploid millet, consisting of 1000 spikelets, there would be formed 850 seeds weighing 10.6 g. In any case, as regards the number of seeds per panicle the diploid form ranks first, while as regards the weight of seeds per panicle the tetraploid form would rank first, if equally fertile. But tetraploid millet is not completely fertile. Lowered fertility in experimental autopolyploidy is, as is well known, a common phenomenon. It is due chiefly to irregularities at meiosis, and such irregularities no doubt occur in our case. However, in tetraploid millet lowered fertility is not expressed to the same degree in all plants. In Table 2 we give counts of the number of normally developed spikelets and the number of seeds for fifteen panicles taken from different plants of the new tetraploid form of millet.

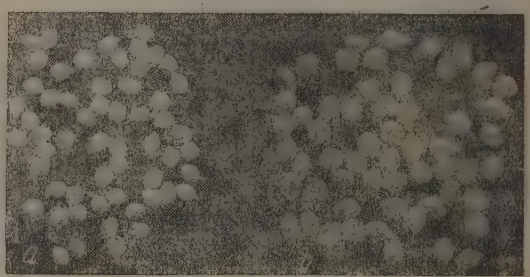

Fig. 3. *a*—control; *b*—tetraploid.

Panicles Nos. 1—14 were taken from plants grown in the field. Panicle No. 15 was taken from Plant No. 11, one of the fifteen plants grown in the greenhouse. These were characterized by low fertility. An exception in this respect was Plant No. 11, which showed the highest fertility of all. As is seen from Table 2, fertility of the various plants ranged from 27.6 to 73.5%. The weight of the yield of individual plants also varied from 5 to 39.8 g, apparently correlating more or less with the degree of fertility. The weight of the seeds of individual control plants also varied, ranging from 20 to 50 g, while one

Table 2

Panicle No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Number of normally developed spikelets	1096	1027	938	778	940	998	807	712	1049	1248	1072	712	932	1031	200
Number of seeds	313	300	300	250	330	340	283	275	420	532	465	310	465	525	147
Fertility (in %)	27.6	29.2	31.1	32.1	34	34	25.1	38.6	40	42.6	43.4	43.5	49.8	50.9	73.5

very bushy plant yielded 70 g. Differences in weight of yield in diploid plants having like fertility are due to differences in amount of tillering.

In 1939 experiments were again made on chromosome doubling in millet by colchicine treatment. As material for these experiments we took seeds of red Saratov 742 (*Panicum miliaceum effusum* Al.). Young plants were treated as described above. Some of the seeds were germinated for cytological analysis. In root-tip cells of the young seedlings we counted 72 chromosomes; in those of the control plants—36.

In addition to the tetraploid millet we thus obtained experimentally, we discovered natural polyploid forms. In looking over seed samples in the Millet Section of the Institute of Plant Industry our attention was attracted by panicles of the pure line No. 2775, which as regards their laxity and the arrangement and size of the spikelets resembled the tetraploid millet we had obtained experimentally. The seeds of this sample, as well as those of sample No. 37, were characterized by larger kernels than those of common millet. Cytological investigation of seedling plants grown from seeds of these two samples confirmed our supposition, for it showed that they had a doubled chromosome number ($2n = 72$), i. e. that these forms were tetraploid. Upon investigating their origin it was found that sample No. 2775 had been obtained in 1930 from the Bezenchuk Experiment Station. In 1937 Lysov and Veselov selected from plants grown from this sample forms which were then reproduced as pure-line material. The second tetraploid form No. 37 was obtained by the Institute of Plant Industry in 1914. The seeds of both samples (Nos. 2775 and 37) are elongated in shape and have red kernels. The panicles of both forms are of the drooping type. Hence, these two forms have not only the same chromosome number but closely resemble each other in type of panicle and size, shape and colour of kernel.

In this connexion it would be of interest to determine more exactly the origin of sample No. 2775. According to Aseyeva, both types (Nos. 2775 and 37) are characterized by lowered fertility and non-crossability with other ordinary forms of millet. The cause probably lies chiefly in their tetraploidy.

The lowered fertility of tetraploid millet may arise from irregularities at meiosis and the formation of quadrivalents. With the aim of overcoming this drawback, it is proposed to cross different tetraploid forms of millet (the two forms obtained experimentally by us belong to two quite different geographical races) and to follow this up by appropriate selection.

Maikop Experiment Station.
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Shuntuk (near Maikop).

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GENETICS

SEEDLING PROGENY OF DIPLOID SPECIES OF POTATO

By R. L. PERLOVA

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The description of new species of potato has usually been based solely on the progeny from asexual reproduction, since many species of potato—wild, as well as cultivated—are characterized by sterility. This has made it impossible to study the seedling progeny of a species and to determine the constancy of the characters by which it is distinguished. Hence, the discovery that many sterile species of potato become fertile when grown in mountainous regions, such as the Pamirs, is of great importance. It is now possible to fill the gaps in the diagnosis of various species of potato and to determine to what extent the characters of the different species are constant.

We have investigated seedling progenies of six diploid ($2n = 24$) species of potato, belonging to three different taxonomic groups: 1) *Commersoniana: Solanum gibberulosum* Juz. et Buk., *S. Horovitzii* Juz. et Buk.; 2) *Aracciana: S. aracc-papa* Juz., *S. catarthrum* Juz.; 3) *Andigena: S. Macmillanii* Buk., *S. cañarense* Buk. Of these six species only *S. gibberulosum* is self-fertile under conditions in Leningrad region, i. e. produces many-seeded berries from natural pollination, while the others are self-sterile under conditions in the central and northern plains of the Soviet Union, particularly in the Leningrad, Moscow and adjacent regions.

When, in 1937, we grew these diploid species at the Khorog Experimental Fields of the Pamir Biological Station of the Tadzhik Branch of the Academy of Sciences of the USSR, at an altitude of 2320 m, they all proved to be fertile. *S. gibberulosum* and *S. catarthrum* were characterized by early maturity of berries (August 4—16), *S. Horovitzii*, *S. Macmillanii* and *S. cañarense* by late maturity (September 16—28), and *S. aracc-papa* by medium-late maturity (September 7—10). *S. gibberulosum* produced a large number of berries, as many as 60—70 per plant, *S. cañarense* and *S. catarthrum*—10—20 berries per plant, and *S. Horovitzii*, *S. aracc-papa* and *S. Macmillanii*—only a very few berries, from 3 to 10 per plant. In the case of the last three species the berries contained only a few seeds (5—10 per berry); in the other three species the berries contained many seeds (100—200 per berry).

When from seeds produced in the Pamirs seedling plants of the six diploid species of potato were grown at the «Krasny Pakhar» Experiment Station (near Leningrad) of the Institute of Plant Industry, it was found that these species differed as regards the ability of their seeds to germinate. Thus, the seeds of *S. gibberulosum* and *S. cañarense*, the species that produced many berries, showed high germinating power (60—80%), while the seeds of the

other four species, not characterized by abundant berry-formation, showed much lower germinating power (8—20%).

Among the investigated diploid species of potato *S. gibberulosum* shows the highest fertility, as is confirmed by the following characters: early maturity (berries formed from the very beginning of the flowering period), abundance of many-seeded berries, very high germinating power of seeds. *S. Horovitzii*, *S. Macmillanii* and *S. aracc-papa* are, on the other hand, only slightly fertile under conditions in the Pamirs, being characterized by late maturity (berries formed only toward the end of the flowering period), few berries containing only a few seeds, and low germinating power of seeds. Of the intermediate group of species *S. cañarense* produced comparatively few berries, but they contained many seeds characterized by high germinating power; *S. catarthrum* also produced many-seeded berries, but the seeds had low germinating power. Naturally, those species whose seeds were characterized by low germinating power were represented by a very small number of seedlings, from 2 to 6. Even in the case of *S. gibberulosum* and *S. cañarense* which produced many-seeded berries with seeds of high germinating power, we only had 25—30 seedlings for our cytological and morphological analyses. Despite the fact that we had such a limited number of seedlings at our disposal, our data are of interest, since they constitute a preliminary characterization of the seedling progenies of several diploid species of potato, seedling progenies of which we were the first to obtain and investigate.

The data given below concern seedling progenies grown at the «Krasny Pakhar» Experiment Station. The cytological analysis, made by P. M. Mamulashvili, showed that all the seedlings of the six investigated diploid species had 24 as their somatic chromosome number. In this case the increase in fertility was not connected with doubling of the chromosome number, as in the case of the triploid *S. Vallis Mexici* when grown under analogous conditions in the Pamirs ([4]). Our morphological analysis revealed that there exists a difference between these diploid species of potato as regards the extent of segregation of characters in seedling progenies of the first generation.

1. *S. gibberulosum* is a wild, Argentine species, distinguished by the following characters: stem much branched; leaves long with many leaflets (4—5 pairs of large leaflets interposed with 3—5 pairs of small leaflets and sometimes, in addition, with 1—3 pairs of very small leaflets, acroscopic or basiscopic); leaflets broad on a long axis; inflorescence branched, many-flowered, with from 15 to 20 flowers per branch, calyx darker green at its base; corolla white, five-lobed, deeply dissected; style long, extending above the stamens for 4—5 mm; tubers white, oval ([3]).

The seedling progeny of this species showed only slight segregation of characters of the vegetative organs, in most cases limited to variation in the number of pairs of large and supplementary leaflets. The very small, acroscopic and basiscopic leaflets were sometimes lacking. Some quantitative characters also varied, such as height of stem and size of leaflets. It is interesting that among the seedling progeny there were segregates with coloured flowers; light violet, light blue, and variegated, i. e. with white-tipped corolla lobes. Of 26 plants 9 had white and the rest coloured flowers. All the seedlings produced berries, but they contained a varying number of seeds (from 20 to 120). The tubers were elongated, white or pink. Hence, in the seedling progeny of *S. gibberulosum* colour of flowers is the character showing the most heterozygosity.

2. *S. Horovitzii* is also a wild, Argentine species, but taxonomically it is far removed from the preceding species. Stem green with characteristic broad, violet crimped ribs; leaves with fewer leaflets than in the preceding

species, i. e., 4—5 pairs of large leaflets and only 1—2 pairs of small, interposed leaflets; leaflets narrow, lanceolate; calyx light green, very short; corolla white, five-lobed, dissected; corolla lobes and tips of lobes narrower than in *S. gibberulosum*; style extending only slightly above the stamens; stigma short; ovary pointed; tubers white, round.

Of this species we had only two seedlings. These varied in the intensity of the green colour of the leaves, the presence or absence of anthocyanin in the axils of the leaflets and leaves, the attitude of the leaflets (drooping or not), the width of the leaflets (1.8—2.2 cm), and the energy and time of flowering, i. e., in secondary characters. The other characters of the species, including sterility under conditions in Leningrad region, were constant. Hence, *S. Horovitzii* is a relatively constant species.

3. *S. aracc-papa* is a wild, Peruvian species from the vicinity of Cuzco. Stem violet; leaves with 5—6 pairs of large leaflets and 1—2 pairs of small, interposed leaflets; large leaflets elliptical, short, widely spaced; interposed leaflets very small, much closer than usual to the pair of large leaflets next below them; calyx bell-shaped, occasionally pubescent; corolla violet; ovary round ([2],[3]).

In the seedling progeny of *S. aracc-papa* we also had only two plants, which differed from each other in the following secondary characters. One seedling had more acuminate leaflets, larger calyx with long tips, and longer style than the other seedling and than the mother plant. It should be noted that *S. aracc-papa* is a fairly polymorphic species, represented by several varieties differing, among other characters, in degree of sterility. Hence of particular interest was the difference between the two seedlings in fertility. One seedling produced a large number of many-seeded berries (as many as 300 seeds per berry). The other, on the contrary, proved to be sterile, and did not produce any berries at all. A microscopic analysis of the pollen showed that the first seedling had 68.6% of normal pollen, while the second had only 37.5%. Moreover, the first seedling had many pollen grains and of large size, while the second had few pollen grains and of small size.

4. *S. catarthrum* is also a wild, Peruvian species from the vicinity of Cuzco. Stem light green, leaves with few leaflets, 3—4 pairs of large leaflets and 1—2 pairs of small, interposed leaflets, calyx broadly bell-shaped; corolla light violet, stigma short, thickened; ovary deeply furrowed ([3]).

The two seedlings of this species differed from the mother plant only in non-essential characters; larger calyx with longer tips, faint violet stripe along the center of each calyx lobe, slightly furrowed ovary. The seedlings proved to be fertile, the berries containing as many as 70 seeds.

In the progeny of *S. aracc-papa* there did not appear the characters of *S. catarthrum*, and vice versa, which shows that these two species are distinct, despite their apparent similarity, that caused Drs. Juzepchuk and Bukasov to consider them at first as one species.

5. *S. Macmillanii* is a Peruvian cultivated species. Stem erect, violet, slightly branched; leaves with few leaflets, 3—4 pairs of large leaflets and 1—3 pairs of small, interposed leaflets; leaflets light green with a thick network of prominent veins, on a short axis; calyx large; corolla violet, large; ovary pyriform; tubers elongated with deep eyes, red.

The seedling progeny of *S. Macmillanii* differed in size of calyx (0.5—1 cm long), colour of tubers (from orange-red to blue-violet), and shape of tubers (from round to elongated). Three out of the five seedlings had white flowers; two seedlings produced berries containing from 50 to 80 seeds.

6. *S. cañarense* is an Ecuadorian cultivated species. Stem erect, light green; leaves with comparatively few, rather narrow, acuminate leaflets, 4—5 pairs of large leaflets and 1—2 pairs of small, interposed leaflets; calyx

bilabial; calyx lobes large, green, with long tips; base of calyx distinctly ribbed; corolla white with a reddish-violet star in center; anthers narrow, short; style slender, extending slightly above the stamens; stigma globular; ovary pyriform-oval with an attenuated tip; tubers yellow oval (1).

The seedling progeny of this species exhibited slight variation in secondary characters, such as presence or absence of anthocyanin in the stem, peduncles or calyx, smooth or notched stigma, style extending or not extending above the stamens. The ovary was narrow, elongated, pyriform or oval. Seven seedlings out of 25 produced berries. The other characters were constant. This indicates that this species is relatively constant. The supposition of Juzepchuk and Bukasov as to the hybrid origin of *S. cañarense*, a putative parental form being *S. goniocalyx*, is therefore but slightly probable.

On the basis of the above data we may group the six diploid species into two categories: 1) those in whose seedling progeny there is observed only slight variation of secondary characters, such as pigmentation in the leaf axils in *S. Horovitzii*, pigmentation of the calyx in *S. cañarense* and *S. catarthrum*, etc., the basic species characters remaining unchanged under condition of sexual reproduction; 2) those in whose seedling progeny there is observed segregation not only of secondary, but also of one or several primary species characters, e. g. white-flowered segregates in *S. Macmillanii*, segregates with coloured flowers in *S. gibberulosum*, fertile segregates in *S. araccpapa*. The species in the first category are practically homozygous, those in the second incompletely heterozygous.

The writer wishes to express her profound gratitude to Dr. S. M. Bukasov for his counsel during the course of the investigation.

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PLANT PHYSIOLOGY

DYNAMICS OF EAR-FORMATION IN SPRING WHEAT *MELANOPUS* 069 IN RELATION TO DEVELOPMENTAL PHASES

By L. S. LUKJANOV

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In the present work our object was to fix the various stages in the formation of ear by spring wheat *Melanopus* 069 and to describe the development of this plant when it is going through its light phase, in terms of morphological characters.

It was not for the first time that we were engaged in studying the dynamics of heading in wheat in relation to phasic development. In 1938 we made similar tests with *Hordeiforme* 0802, a late spring variety. Then it was found that the need of wheat plants in long days was closely linked with the process of heading. The interval when the plants responded to daylight stretched from the time of growing point elongation to the initiation of anthers in fourths flowers of the middle spikelets.

It may be remarked that there is a great scarcity of literature on the dynamics of heading in general, and with reference to phasic development the subject has only begun to be treated.

In 1939 we started an experiment with spring wheat *Melanopus* 069. This time the formation of ear was investigated in its various stages more thoroughly, and reference to phasic development was made. Planting was done in containers on April 18. On April 25 the plants appeared above ground in mass.

In order to disclose their light phase the plants were given natural and short days according to plan (see table). Those of the first set were started on short (10-hour) days when they emerged from the soil. In the second and third set the plants were transferred to the short daylength three and six days later, respectively. From then on the interval between one set and the next was 2 days. In that way new sets of plants were constantly introduced in the experiment, until the controls started heading in mass. When this occurred, all plants were transferred to natural daylight.

Our anatomical and morphological investigation of the ear of spring wheat *Melanopus* 069 has shown that its formation runs through a series of stages which are connected with the developmental phases of the plant.

Sets	Number of natural days elapsed between mass germination and start of short-day treatment	Morphological stage of plants at the start of short-day treatment			Interval between germination and heading in days	At the time of harvesting		
		Number of leaves	Length of culm in cm	Length of spike on the main stem		Length of spike on the main stem in cm	Length of the base internode in the main stem spike in cm	Number of spikelets in the main stem spike
1	0	1	—	—	64	6.68	0.46	20.60
2	3	2	—	6 divisions*	64	6.33	0.45	20.90
3	6	—	—	—	63	6.98	0.46	21.60
4	8	3	—	8 divisions	62	10.89	4.48	22.46
5	10	3	—	12 »	62	20.18	13.78	23.10
6	12	4	—	17 »	59	19·30	12.25	17.26
7	14	—	—	—	56	17.48	10.00	15.00
8	16	—	—	34 divisions	55	9.48	2.90	16.00
9	18	5	—	46 »	54	8.68	1.65	15.50
10	20	5	0.2	73 »	53	8.26	1.18	14.80
11	22	6	0.3	82 »	52	7.11	0.78	16.00
12	24	6	0.8	123 »	47	6.35	0.55	14.86
13	26	7	2.7	185 »	45	6.34	0.47	15.80
14	28	7	5.3	6.7 mm*	44	6.31	—	16.00
15	30	8	8.0	—	43	6.11	0.42	15.50
16	32	8	9.9	16 mm	43	5.99	0.42	15.50
17	34	8	13.1	32 »	42	5.95	0.42	15.30
18	36	8	19.2	—	42	5.86	0.42	15.00
19	38	8	23.7	58 mm	42	5.90	0.42	15.20
Control	40	8	26.8	63 »	42	5.86	0.42	15.30

As is obvious from the table, there was nearly no change in the size and form of the growing point during the first 6—7 days after germination; nor did these plants show any significant deviation from set 1 as regards time of heading. Spike length was also little affected in the plants transferred to short daylength on the day of mass germination, as compared to the control, but their number of spikelets per spike increased considerably (in the 2nd and 3rd sets).

The days that followed (from 6th to 10th) were characterized by a rapid elongation of the growing point. In the experimental plants the external morphological character of this phase was the appearance of the 3rd leaf. If the plants were transferred to short daylength during this period, they greatly delayed heading (for 20 days), increased in spike length 5—14 cm, compared to controls, and had 7—8 spikelets more per spike. It may therefore be assumed that the period of growing point elongation marks the beginning of the light phase in development.

The next stage in the formation of the spike appears to begin with its differentiation into spikelets. In that period, on the surface of the growing point, at its base and farther up, there appeared the primordia of spikelets in the form of semicircular beads inclined at a nearly right angle, while the leaf beads were inclined more obliquely. The external morphological character of this period was the appearance of the 4th leaf. Plants that had

* Spike length was measured under the microscope and is expressed in the table either as the number of divisions of the eyepiece micrometer (set 2 to 13) or in mm (sets 14 to control).

undergone short-day treatment from that time onwards were 20 days behind the controls in heading, and their spike had strongly increased in length owing to a strong growth of its base; the number of spikelets in a spike had increased, too. This was probably the critical period for the plant tested when its long day requirement was the highest.

Differentiation into spikelets was complete in 7—8 days and the spike entered a new phase, the formation of elements in the spikelets located in its nearly lowermost part. On these spikelets there were forming processes, the primordia of sterile scales, under which the lower floral scale was set. In this very period the lower internode between the 4th and 5th leaf started spreading in length and the expansion of the 5th leaf was going on.

The plants now transferred to short days would delay heading for 11 days and bear longer spikes compared to the control. But the number of spikelets per spike was not higher, a confirmation to the fact that the spike had completed its differentiation into spikelets when the short-day treatment began.

The anthers form very early. In the first flowers of the middle spikelets (the 6th, 7th, 8th and 9th from below) anthers were found on the 24—25th day after germination. Toward that time the 6th leaf made its appearance, and the stem was rapidly growing owing to extension of its 1st and 2nd internode.

When full expansion of the 7th leaf blade was taking place, the spike continued elongation. In the main stem spike the middle spikelets were almost completely differentiated into florets which amounted to 6 in number. Of these two lower florets were already bearing anther primordia which could easily be separated from them. The stem reached 5.5 cm in length; this would mean that the plants proceeded to culm-formation.

Such plants when transferred to short daylengths delayed heading for 2 or 3 days compared to the control, and bore only slightly longer spikes at the end of the growing period.

All of this testifies to a need of plants for long days in that period, too.

With the appearance of the 8th leaf (the last in our variety), on the 30th day since germination, the middle spikelets of the main stem spike had their anthers perfectly formed in the first, second and third florets. The anthers were set also in the fourth florets, but unlike those in the first three, of which each one could easily be detached and put down in a separate drawing, they were only crushed when pressure was exerted on the spikelet.

No pollen could yet be detected in the anthers on the 30th day. The plants taken to short daylength at this stage, were 1 day behind the controls in heading and but slightly surpassed them in spike length towards the end of the growing period. Plants subjected to short photoperiods 4 days later differed from the controls neither in the date of heading nor in the length of spike, although some growth had been made by the latter on short photoperiods. It will be obvious that in our plant the light phase is complete in 30—31 days following the germination.

After this period had elapsed, the spike continued growing, the stamens and pistils made rapid progress. The pistils increased in length at a particularly high rate, and fertile pollen was formed in them 5—6 days before heading, their stigmas grew longer. The stems increased in height owing to elongation of their 4th internode, and further growth during heading was due to the 5th internode which is the last in this wheat.

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PLANT PHYSIOLOGY

PERIODICAL SUPPLY OF NITROGEN TO SPRING WHEAT

By A. KALTIA

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The object of this work is so far as possible to establish the critical period in the need of spring wheat for nitrogen; or, in other words, to find out the time within its growing season when the plant is the most sensitive to this element and responds to it with increasing the yield to a maximum.

The experiment was made with the variety *Lutescens* 062, according to the plan represented in the table. All tests were made in duplicate, the method of water cultures in Hellriegel's nutrient solution being used. The vessels were of 2.5 liter capacity and contained 3 plants each.

When calcium nitrate was eliminated from the nutrient solution, it was replaced by calcium sulphate. The sets VII, VIII and IX were controls.

On the 15th day of vegetation the plants without nitrogen were already distinguishable by their outward appearance from those on full mixture. And on the 21st day their leaves began withering as a result of starvation. The nitrogen starvation of these plants manifested itself in several other signs quite alike to those which had previously been placed on record by Vorobiev ([1]), Sabinin ([14]), Ressel ([13]) and others. It was also responsible for a delay in the formation of some of the leaves. The 5th leaf formed 8 days later in the plants without nitrogen than in the controls, and their 6th leaf formed 9 days later.

There was no tillering among starving plants. And even if they had been transferred onto the full nutrient mixture as early as in the 6th decade of vegetation, tillering started when the main stem was already dry; in the plants transferred onto a nitrogen free solution the young shoots began drying.

Respecting the effect of a periodical supply of nitrogen on tillering, it may be inferred from the tabulated data that: 1) If the plant is started with nitrogen only to receive it during the first 13 days, no tillering will ever take place (set I in the table; vessel 1 in Fig. 1); but if it has grown for the first 13 days without nitrogen and has been given later a full supply (set X in the table, vessel 23), tillering is even favourably affected (compared to the control). 2) If the plant grows without nitrogen from the third or fourth decade onwards, tillering will sharply decrease and the number of productive shoots will be especially low (vessel 3).

Intensity of tillering determines the formation of the above-ground organic mass of the plant; this is also obvious from the table. In that respect we may say that:

1) The plants supplied fully during the first 13 days of vegetation, but deprived of nitrogen afterwards show only a slight addition to the organic mass of the plants which received no nitrogen throughout their growing period (sets I and XVII; vessels 1 and 37).

2) With no nitrogen during the first 13 days, the decrease in dry mass is not considerable compared to the control (sets X and IX).

After five decades no further supply of nitrogen will increase the organic mass (set V). Nitrogen has a strong influence on the development of the root system. Both the table and Fig. 1 (59th day of vegetation) show this.

According to Domontovich and Groshenkov ([5]), a disturbed supply of nitrogen affects the root system unfavourably. Miller ([3]) points out that the pH of the medium is also a factor here. Several different investigators, as Ovechkin ([10]), Dittmer ([3]) and others, have found that not only does the root system affect the whole life activity of the above-ground part, but it is also itself liable to be affected by the latter.

Under conditions of nitrogen starvation this relation between the above-ground parts and the roots is reduced, as is seen from the table. This is because absence of nitrogen in the nutrient solution more strongly checks the accumulation of matter in the above-ground parts than in roots.

Effect of Nitrogen Supplied to Spring Wheat
(average data per vessel

Sets	Experimental plan		Length of vegetation in days	Stems		Number of spikelets per spike
	With nitrogen from the start of vegetation during	Without nitrogen from the start of vegetation during		Total number	Spike-bearing	
I	13 days	—	79	3	3	9.7
II	23 »	—	79	6	4.5	10.3
III	33 »	—	79	10	9	11.8
IV	44 »	—	79	18	18	12.7
V	54 »	—	79	17.5	17	11.4
VI	66 »	—	79	17.0	16	10.6
VII	81 »	—	81	19.5	19.5	10.2
VIII	81 »	—	81	17.0	17.0	11.8
IX	81 »	—	81	18.8	18.8	10.9
X	—	13 days	87	19.5	19.5	10.4
XI	—	23 »	98	16.5	16.5	8.3
XII	—	33 »	98	8.0	8.0	5.9
XIII	—	44 »	98	4.5	4.5	5.1
XIV	—	54 »	81	3.5	3.5	7.1
XV	—	66 »	81	3.0	3.0	7.3
XVI	—	81 (throughout)	81	3.0	3.0	7.0
XVII	—	81 (throughout)	81	3.0	3.0	7.3

The works by Vostokov ([2]), Dominikovsky ([4]), Molchanov and Shyrshov ([3]) and others have brought to light the powerful influence of mineral supply on physiological processes in plants, particularly on stomatal aperture, on the transpiration coefficient and on other functions of the leaf. Vostokov, for instance, has shown that with excessive nitrogen in the manure, the plant loses drought resistance and that this effect is offset by addition of phosphorus. He believes that the plants do best when the ratio of nitrogen to phosphorus in the nutrient solution is 30 to 90. Udolskaya ([13]) writes that the effect of phosphorus is to increase the water holding power in plants. Their plasma becomes more retentive of water, and so a normal assimilation is secured under conditions of water deficiency.

By intensifying the development of the above-ground mass nitrogen is indirectly responsible for an increase in the coefficient of transpiration. This is brought out by the tabulated data. High transpiration coefficients occurred among the plants of the XI, XII and XIII sets. This was probably because the temperature of the air was high and its humidity low, when these plants were tillering after being transferred to the full nutrient solution.

The nutrition of plants has also a strong influence on the development of reproductive organs. This has been stated by a number of workers [([6], [11], [15], [16]) and others]. Our experiment has furnished similar evidence (see Fig. 2 and the corresponding data in the table). From these data it is evident that the influence of nitrogen on the reproductive development in spring wheat, especially on the formation of spikelets, is decisive during the first 4 decades of vegetation, when the spikelets are forming (the light phase). It should be noted that nitrogen from the nutrient solution is not absorbed by the plant at the same rate all through the growing period.

According to Turchev ([17]), the uptake of nitrogen by the winter rye is a maximum in the fall and especially at the early phases of growth during 10—20 days following the winter. As for the spring wheat, Miller ([8]) found

a t Different Times within Growing Period
containing 3 plants)

Air-dry matter						Transpiration coefficient	
Above ground		Roots		Total			
in gs	% in total	in gs	% in total	in gs	% to control	Without roots	With roots
0.802	65.4	0.440	34.6	1.24	5.6	1 230	903
5.742	81.6	1.465	18.4	7.05	31.8	581	478
14.282	89.7	1.647	10.3	15.93	72.0	464	416
19.275	90.3	2.035	9.7	21.34	96.5	491	444
20.535	91.0	2,030	9.0	22.57	101.9	533	485
19.442	86.7	2.995	13.3	22.44	101.4	666	677
—	—	—	—	—	—	—	—
19.139	86.5	3.070	13.5	22.21	100	681	586
—	—	—	—	—	—	—	—
17.115	84.6	3.114	15.4	20.23	91.4	629	533
6.700	84.6	1.250	15.4	7.95	35.9	753	635
1.365	76.7	0.415	23.3	1.78	8.3	1 018	782
0.850	74.2	0.300	25.8	1.15	5.2	917	681
0.417	72.3	0.160	27.7	0.58	2.6	525	362
0.342	65.5	0.180	34.5	0.52	2.3	1 079	707
0.360	67.3	0.175	32.7	0.54	2.4	555	374
0.425	64.4	0.235	35.6	0.66	3.0	701	452

it to absorb the highest amounts of nitrogen when it was forming ears and started flowering, whereas Rippel (7) reported an early absorption of nitrogen, compared to the other elements. According to the latter, when the plants have brought their store of nitrogen to 78.9% of the amount contained in an adult plant, they have only 36.1% of calcium. And what is more, he found 75% of total nitrogen in plants stored with only 8.9% of dry matter. Ovechkin and Burian (2) reported daily variations in the absorption of nitrogen. They found it to be far more intensive by day than by night.

No data on the uptake of nitrogen by the plants in our experiment will be given here except to say that: 1) with full supply the absorption of nitrogen was higher in the 3rd and 4th decade of vegetation, and its assimilation was also a maximum at that time; 2) much more nitrogen was absorbed by young plants than by old, and as the plants grew older they took up nitrogen in ever decreasing amounts; 3) the curve of nitrogen content in plants went steadily up all through the growing period but for the last decade, when a decline was observed.

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PLANT PHYSIOLOGY

HORMONES OF CELL DIVISION IN COTTON

By K. SUKHORUKOV and V. FILIPPOV

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The active rôle of cell division hormones in the growth process of higher plants admits of no doubt at present. Recent years have brought many new facts to bear on the mechanism underlying the participation of these hormones in cellular processes, namely in those of metabolism, in which they appear to be involved by entering the enzymes and enzymic systems ([10], [11]). Their very name of cell division hormones has certainly become a somewhat conventional term, since they are not confined to the cells of meristematic tissues, but occur in the differentiated tissues as well, though in different quantities.

While taking part in cellular processes these hormones can also be laid up and preserved in the form of some unknown combinations with protein ([7],[9]) and so we have to speak of cell division hormones in a free and in a bound condition; the first are easily extracted, whereas the second can be isolated only through disintegration of the protein.

Our investigation into the subject was made on cotton, since this plant seemed best to suit the intended purpose as it has a long period of growth and fruit-formation, is very sensitive to external influence and stores up much reserve matter in its stem.

The experiments were set up and the material collected at the experimental station for cotton and lucerne in Fergana in the summer of 1939. The variety tested was *Gossypium hirsutum* 8517. Attention was mainly directed to the dynamics of the bios matter and vitamin B_1 in the stems and fruits. Both the bios and vitamin B_1 were extracted from dried plants by 80% alcohol on heating for half an hour over a boiling water bath; the alcohol was removed from the extracts by evaporation and the residue dissolved in distilled water. The yeast method was used in the estimation of bios, while vitamin B_1 with vitamin-pyrimidin and vitamin-thiazol were determined according to Schopfer. The hormone solutions were introduced into nutrient media for the yeast and *Phycomyces nitens* in such proportion that their content in 0.01 g of plant's dry matter corresponded to 1 cm^3 of nutrient medium. All determinations were made in quadruplicate. The amount of hormones was expressed in relative figures by putting the quantity of yeast cells and the dry weight of mycellium in control media at one.

Weight Variation in the Stem of Cotton During Flowering. Cotton, like woody plants, stores up considerable amounts of reserve matter in its stem. This was shown microchemically for the cotton from Fergana plain in the works by Kling ([3],[4]), in which a number of regu-

larities in the storage and consumption of starch were established. The store of reserve matter in the stem grows with the assimilative activity of the plant and goes down in the period when increased amounts of organic matter are consumed. In other words, the mass of the stem in a growing plant has its variation determined by the changes in the amount of plastic substances and structural elements. When the amounts of cell division hormones are to be calculated on a relative basis, it is well to estimate the changes in plastic substances quantitatively, since they will to a certain extent bring out the dependence between the consumption of plastic substances and hormones. Let us refer here to some data on the change in absolute dry weight of the stem of an average plant among 20, while in the period of mass flowering and fruit-formation. The determinations were made at the end of July and in the first half of August, the plants being taken from field plots, when part of them had been deprived of light by screening during 15 days. Part of the plants had their buds, flowers and fruits removed since the 25 of June. Here are these data.

	Date of samp ing / Abs. dry weight of a stem			
Plants under normal growing conditions		July 24 16.08	Aug. 6 15.07	Aug. 25 26.02
Screened plants	Do	Aug. 1 15.79	Aug. 10 13.86	Aug. 14 7.71
Plants deprived of their buds, flowers and fruits	Do	July 24 23.94	Aug. 6 40.27	Aug. 15 48.22

The figures have no claim to great accuracy, but they give a general picture of the changes in the stem mass. If the curve of the stem weight shows a decline at the beginning of August in normal plants, this is not a matter of chance, but is probably due to enhanced flowering and fruit-formation at this period. Screening reduces the weight of the stem to a half in a fortnight; the leaves begin growing yellow and fall off, as do the buds and young bolls. When the fruiting elements are removed, the stem increases in weight more than two times in 20 days; the plants form new sympodia, the main stem makes strong growth and many new leaves appear. From comparing these figures, it may be concluded that a considerable portion of the reserve material stored by the stem is used up in the formation of flowers and fruits, and the assimilation activity of the plant cannot offset the total expenditure of organic matter.

Bios and Vitamin B_1 in the Stems and Fruits. By the method mentioned above determinations of bios and vitamin B_1 with vitamin-pyrimidin and vitamin-thiazol were made in the stems and fruits of an average sample. All fruits were of medium size. In Table 1 the contents are given in relative magnitudes, the number of yeast cells and the dry weight of *Phycomyces nitens* mycellium in the media not added with extracts being put at one.

Comparisons of these figures lead to certain conclusions. The stem was rather high both in bios and vitamin B_1. And it increased in each of these two hormones by nearly 20% in 28 days following the removal of flowers. But this increase was not attended by a corresponding increase in weight of the whole stem. We refrain for the time being from establishing the relations between the amount of hormones and that of mobile reserve matter. Screening had a decreasing effect on the content of hormones in the stem, especially of vitamin B_1 which went abruptly down without light; the content of bios under these conditions showed fluctuations which will be interpreted later. Light was withheld from the plants for a long interval during which they lost the bulk of their fruiting elements and leaves, and their stems

Table 1

Content of Bios and Vitamin B_1 in the Stems and Fruits of Cotton

Conditions of growth and parts tested	July 27		Aug. 15		Aug. 20		Notes
	Bios	Vitamin B_1	Bios	Vitamin B_1	Bios	Vitamin B_1	
	Normal plants						
Stems . . .	3.10	101.0	2.30	—	6.80	102.1	
Fruits . . .	10.00	125.0	7.70	88.0	8.60	83.0	
	Plants with removed flowers and fruits						
Stems . . .	3.00	—	—	—	8.70	121.2	Removal of flowers was begun June 25
	Screened plants						
Stems . . .	5.50	86.0	9.40	48.0	4.10	18.9	Screening started 5 days before first sampling
Fruits . . .	5.90	111.0	2.60	104.0	1.70	114.0	

decreased in cell division hormones. So the latter appear to form mainly in the assimilating leaf, since their uptake through the root in significant quantities is not possible in the case of a cotton growing on such soils as those of Fergana plain, according to Bonner and Greene ([3]) who have tested a series of plants for vitamin B_1. Another fact inviting attention is the comparative constancy of the content of vitamin B_1 in fruits, while the content of free bios in them goes down day by day during screening. The fact has already been placed on record by several investigators. Leong ([12]) has found the kernels of wheat and barley to show no variation in vitamin B_1 in response to different cultivation and manuring of the soil. Similar observations have been reported by Scheunert and Wagner ([13]). In different varieties of wheat quantitative fluctuations of vitamin B_1 in the kernel are very small, according to Murri ([5]).

Variations of the bios content in the leaves of woody plants during the growing season, in a course of 24 hours, at the time of leaf fall and with a change in weather conditions have been recorded by Gavrilov ([2]). Rakitin and Yarkovaya ([6]) have also observed considerable changes in the amount of free bios in various parts of a plant at the time of leaf fall. The plants increase in free bios either at the expense of protein breakdown within the cell ([1]) or through synthesizing it in their leaves ([7]). The variations between cellular proteins and cell division hormones are of certain interest indeed, but the question cannot be settled on the basis of the available data. We have made an attempt to find out the ratio of the free hormones to those combined with protein in the stems. For estimation of combined hormones, the stems were dried, ground and treated during 3 days with pepsin and hydrochloric acid; the free hormones were preliminarily extracted with 80% alcohol, as has been stated above. Various stem samples were taken from a single cotton variety No. 8517. In Table 2 the results of analysis are given in relative figures.

From these figures it is obvious that in face of a very wide range of variations exhibited by the bios (0.5—3.3), the ratio for vitamin B_1 is fairly constant (1.7—2.2). Forbearing any further interpretation of the results obtained, we nevertheless think it possible at this juncture to explain the

Table 2

Cell Division Hormones, Free and Bound, in the Stem of Cotton

Samples	Bios		Ratio of free to bound	Vitamin B_1		Ratio of free to bound
	Free	Bound		Free	Bound	
I	5.65	1.70	3.3	128.8	64.8	2.0
II	2.20	3.15	0.7	149.6	66.4	2.2
III	3.95	7.55	0.5	83.2	48.0	1.7

strong fluctuations in the content of bios by its transition from one condition into another.

With this we arrive at the following conclusions. Along with laying up of plastic substances the storage of cell division hormones—bios and vitamin B_1—takes place in the stems of cotton. The site of their formation is the assimilating leaf. In the absence of photosynthesis the hormones stored in the stem are used up by the plant in its life-process. The outflow of vitamin B_1 from the stem is especially rapid. As the stem grows lower in cell division hormones, it loses its leaves and fruiting elements. The formed fruits (bolls) have a constant content of vitamin B_1, and this seems to be the case with the fruits of other plants, too. The amount of bios stored in the fruit decreases appreciably, if no new bios is formed by the plant.

Estimations of free bios and of that combined with protein have shown that the ratio of the former to the latter is liable to considerable variation, while the respective ratio for vitamin B_1 is comparatively constant.

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PHYTOPATHOLOGY

PEA VARIETIES WITH PODS RESISTANT TO *ASCOCHYTA PISI* LIB.

By V. K. ZAZHURILO

(Communicated by A. A. Richter, Member of the Academy, 17.VI.1940)

The disease caused in peas by *Ascochyta pisi* Lib., a kind of fungus, is among pests which can be controlled effectively only by cultivation of resistant varieties. Up to the present no such variety of peas has received any important extension in agricultural practice. Meanwhile, according to literary data, resistant varieties can be obtained out of strongly susceptible ones (for instance, Victoria Heine) by means of simple selection. This of course is a costly procedure and a great waste of time. It is far more reasonable first to look among the varieties available for such as are even now relatively if not absolutely resistant to *Ascochyta pisi*.

As the affected seeds are largely responsible for the recurrence of the disease in spring, it is advisable in the first place to test the resistance of pods, through whose valves the fungus penetrates to the seeds. Sure enough, the varieties with immune or but slightly susceptible pods will suffer less from the disease in question, even if their leaves and stems are susceptible.

As I know, Riakhovsky was the first to point out the practical importance of such partial resistance of pea varieties. In his communication he has mentioned some varieties from Ramon breeding station with weakly susceptible pods. The varying resistance of pods in different peas has been reported also by Bondartseva-Monteverde and Vassilievsky.

Our tests on the affection of various pea varieties with *Ascochyta pisi* were carried out at the steppe variety-plot and during stationary variety tests at the Kamennostepnaya breeding station in 1935. On the whole, there were about 100 varieties of most diversified origin under our observation.

The results of affection were summed up two times during the season, for the vegetative parts, at the period of pod formation; for pods, 2—3 days before harvest. The inspection of every variety extended over 200 plants and 400 pods divided into 10 samples, whereby there were two identical sets. Affection was estimated after a 5-point scale of spottedness. The results are set forth in the table. They refer only to the varieties tested at the steppe variety-plot. In all varieties the vegetative parts were very liable to the attacks of *Ascochyta pisi*. But there were striking differences with regard to the affection of pods. These differences were by no means a matter of chance, since they were observable in every sample and recurred in the duplicate set. Along the varieties whose pods were affected strongly (Victoria Heine, Victoria Strube, Fodder Grep, etc.) there were varieties with weakly affected pods (Folger Heine, Waxy 019, Mench etc.). In them the ratio of diseased

Variety	Affection of plants (before harvest)		Affection of pods (before harvest)	
	Ratio of affected in %	Progress of disease in %*	Ratio of affected in %	Progress of disease in %*
Victoria Strube (standard)	100	34.8	46	30.8
Grep (fodder)	100	34.8	52.7	19.2
Victoria Podolskaya	100	33.6	43.7	16.0
Victoria Heine (standard)	98	35.6	47.0	15.9
Victoria Uladov, Brand I	100	34.4	43.5	14.8
Capital	100	33.3	36.5	13.0
Victoria pink 079	100	34.5	30.7	12.1
010 Buss	100	33.3	29.0	10.1
Victoria Mendorf (standard)	100	35.1	42.0	9.2
Victoria Uladov, Brand III	100	33.3	24.5	8.2
Ambrosia	100	33.3	16.5	5.5
Victoria of Jena	100	33.3	15.3	5.4
Victoria green 080	100	33.3	9.0	4.3
Teredag	98.5	32.8	6.5	2.1
Manshold	89.5	29.5	4.0	1.6
Solo	100	33.3	4.0	1.3
Folger Heine	98.5	31.5	3.5	1.1
Waxy 019	91.5	30.4	2.7	0.9
Concordia	98.5	32.8	2.5	0.9
Mench	100	33.3	2.8	0.7

pods was under 5%. And what is more, the degree of affection was very slight (isolated small spots); this is especially important for obtaining a sound seed material. It would appear as if the parasite had found no favourable conditions for its development on pod valves of certain varieties. But there is also another reason to account for slight affection of pods in these peas. And that lies in a possible chance coincidence of their flowering- and ripening-time with the period unfavourable for infection. If so, the differences observed were due to purely external causes. But this assumption does not hold water, for some of the strongly affected varieties have passed their flowering- and ripening-period simultaneously with the slightly affected ones. The slight susceptibility of pods to the attacks of *Ascochyta pisi* can therefore be regarded as a varietal character.

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* Progress of disease is estimated as the ratio of the average to the highest value and its figures therefore reflect not merely the degree of affection, but also the spreading of the disease.

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